Answers (contd)

(16) $\frac{5}{2(x-1)} + \frac{11}{4}\log\left|\frac{x-3}{x-1}\right|$ (17) $\frac{2}{17}\log|2x+1| - \frac{1}{17}\log|x^2+4| + \frac{1}{34}\tan^{-1}\left(\frac{x}{2}\right)$

(18) $\frac{5}{26}\log\left|\frac{(3\log x+2)^2}{(\log x)^2+1}\right| + \frac{12}{13}\tan^{-1}(\log x)$

Type-4 : Integration by parts

$\boxed{\text{Subtype- A}}$

Integrate w.r.t. x

(1) $x^2 \cdot \log x$ •(2) $x \cdot \tan^{-1}x$ (3) $\frac{x}{1+\cos 2x}$ •(4) $\frac{x}{1-\sin x}$

(5) $x \sin 2x \cos 5x$ •(6) $x\cos^3 x$ (7) $x^2 \cdot 5^x$ (8) $x^2 \cdot \sin 3x$

•(9) $x^3 \tan^{-1}x$ (10) $x^2 \cos^{-1}x$ (11) $\cos\sqrt{x}$ •(12) $e^{\cos x}\sin 2x$

Answers (Add a constant c to all answers)

(1) $\frac{x^3}{9}(3\log x - 1)$ (2) $\frac{x^2}{2}\tan^{-1}x - \frac{1}{2}(x - \tan^{-1}x)$ (3) $\frac{1}{2}\left[x\tan x - \log|\sec x|\right]$

(4) $x(\sec x + \tan x) - \log|\sec x(\sec x + \tan x)|$

(5) $\frac{x\sin 3x}{6} - \frac{x\sin 7x}{14} + \frac{\cos x}{18} - \frac{\cos 7x}{98}$

(6) $\frac{1}{4}\left[\frac{x\sin 3x}{3} + \frac{\cos 3x}{9} + 3x\sin x + 3\cos x\right]$

(7) $\frac{5^x}{\log 5}\left[x^2 - \frac{2x}{\log 5} + \frac{2}{(\log 5)^2}\right]$ (8) $-\frac{x^3}{3}\cos 3x + \frac{2x\sin 3x}{9} + \frac{2\cos 3x}{27}$

(9) $\frac{1}{4}(\tan^{-1}x)(x^4-1) - \frac{1}{12}(x^3 - 3x)$ (10) $\frac{x^3}{3}\cos^{-1}x - \frac{1}{3}\sqrt{1-x^2} + \frac{1}{9}(1-x^2)^{3/2}$

(11) $2\left[\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x}\right]$ (12) $-2e^{\cos x}(\cos x - 1)$

$\boxed{\text{Subtype-B}}$ $\int u \cdot v \, dx$ where $u \in I, L$ & $v = 1$

Integrate w.r.t. x

(1) $\sin^{-1}x$ •(2) $\tan^{-1}x$ (3) $\frac{\log(\log x)}{x}$ •(4) $(\log x)^2$ (5) $\sin x \cdot \log(\cos x)$

•(6) $\log(x^2+1)$

<u>Answers</u> (Add a constant c to all answers)

(1) $x\sin^{-1}x + \sqrt{1-x^2}$ (2) $x\tan^{-1}x - \frac{1}{2}\log|1+x^2|$

(3) $\log x\left[\log(\log x) - 1\right]$ (4) $x\left[(\log x)^2 - 2\log x + 2\right]$

(5) $\cos x\left[1 - \log(\cos x)\right]$ (6) $x\log(x^2+1) - 2\left[x - \tan^{-1}x\right]$

$\boxed{\text{Subtype-C}}$ Problems on $\int e^x\left[f(x) + f'(x)\right]dx = e^x \cdot f(x) + c$

Integrate w.r.t. x

(1) $e^x\left[2 + \cot x - \operatorname{cosec}^2 x\right]$ •(2) $e^x\left(\frac{1+\sin x}{1+\cos x}\right)$

(3) $\frac{xe^x}{(x+1)^2}$ •(4) $e^x\left[\frac{(x+2)}{(x+3)^2}\right]$ (5) $e^x\left[\frac{x^2+1}{(x+1)^2}\right]$

•(6) $e^{5x}\left[\frac{5x\log x + 1}{x}\right]$ (7) $e^{\tan^{-1}x}\left(\frac{1+x+x^2}{1+x^2}\right)$

(8) $\frac{\log x}{(1+\log x)^2}$ •(9) $\log(\log x) + \frac{1}{(\log x)^2}$

<u>Answers</u> (Add a constant c to all answers)

(1) $e^x(2+\cot x)$ (2) $e^x \tan\frac{x}{2}$ (3) $\frac{e^x}{x+1}$ (4) $\frac{e^x}{x+3}$

(5) $e^x\left(\frac{x-1}{x+1}\right)$ (6) $e^{5x}\log x$ (7) $x \cdot e^{\tan^{-1}x}$ (8) $\frac{x}{1+\log x}$

(9) $x\left[\log(\log x) - \frac{1}{\log x}\right]$

(81) $\dfrac{1}{x^2+8x+12}$ • (82) $\dfrac{\sec^2 x}{3\tan^2 x+5\tan x+4}$ (83) $\dfrac{e^x}{\sqrt{2e^{2x}-5e^x-1}}$

(84) $\dfrac{1}{4-5\cos x}$ (85) $\dfrac{1}{2-3\sin x}$ • (86) $\dfrac{1}{3-2\sin x+5\cos x}$

(87) $\dfrac{1}{2\sin 2x-3}$ • (88) $\dfrac{1}{5+4\cos 2x}$ (89) $\dfrac{1}{(x-3)(x+5)}$

(90) $\dfrac{5x+2}{x^2-3x+2}$ (91) $\dfrac{x^2}{(x+3)(x+2)}$ • (92) $\dfrac{x^3+5x-8}{x^2-5x+4}$

(93) $\dfrac{2x^2-3}{(x^2-5)(x^2+4)}$ (94) $\dfrac{2x}{(x^2-1)(x^2-3)}$ • (95) $\dfrac{\sin 2x}{\cos^2 x+4\cos x+3}$

(96) $\dfrac{5x^2-1}{x(x^2-1)}$ (97) $\dfrac{1}{2\cos x+\sin 2x}$ (98) $\dfrac{1}{\log x^x\left[(\log x)^2-3\log x+2\right]}$

• (99) $\dfrac{1}{\sin x(3+2\cos x)}$ (100) $\dfrac{x+5}{x^3+3x^2-x-3}$ (101) $\dfrac{1}{(2x-1)(x^2+9)}$

• (102) $\dfrac{5e^x}{(e^x+1)(e^{2x}+9)}$ (103) $x\sin 5x$ (104) $x^3\cdot\log x$

• (105) $\dfrac{x}{1-\cos 2x}$ (106) $x\cos 3x\cos x$ • (107) $x\sin^2 x$

(108) $x^2 e^{3x}$ (109) $x^2\cos x$ • (110) $x^2\tan^{-1}x$

(111) $x^2\sin^{-1}x$ (112) $\sin\sqrt{x}$ • (113) $\cos\sqrt[3]{x}$

(114) $\cos^{-1}x$ (115) $\cot^{-1}x$ (116) $\cos x\cdot\log(\sin x)$

• (117) $\log\left(x+\sqrt{x^2+a^2}\right)$ (118) $e^x\left(\dfrac{1}{x}-\dfrac{1}{x^2}\right)$ (119) $e^x\left(\dfrac{2+\sin 2x}{1+\cos 2x}\right)$

Answers (contd)

(104) $\frac{x^4}{4}\left[\log x-\frac{1}{4}\right]$ (105) $\frac{1}{2}\left[-x\cot x+\log|\sin x|\right]$

(106) $\frac{x\sin 4x}{8}+\frac{x\sin 2x}{4}+\frac{\cos 4x}{32}+\frac{\cos 2x}{8}$ (107) $\frac{x^2}{4}-\frac{x\sin 2x}{4}-\frac{\cos 2x}{8}$

(108) $\frac{x^2e^{3x}}{3}-\frac{2xe^{3x}}{9}+\frac{2e^{3x}}{27}$ (109) $x^2\sin x+2x\cos x-2\sin x$

(110) $\frac{x^3}{3}\tan^{-1}x-\frac{x^2}{6}+\frac{1}{6}\log|1+x^2|$ (111) $\frac{x^3}{3}\sin^{-1}x+\frac{1}{3}\sqrt{1-x^2}-\frac{1}{9}(1-x^2)^{3/2}$

(112) $2\left[-\sqrt{x}\cos\sqrt{x}+\sin\sqrt{x}\right]$ (113) $(3x^{2/3}-6)\sin\sqrt[3]{x}+6\sqrt[3]{x}\cos\sqrt[3]{x}$

(114) $x\cos^{-1}x-\sqrt{1-x^2}$ (115) $x\cot^{-1}x+\frac{1}{2}\log|1+x^2|$ (116) $\sin x\left[\log(\sin x)-1\right]$

(117) $x\log(x+\sqrt{x^2+a^2})-\sqrt{x^2+a^2}$ (118) $\frac{e^x}{x}$ (119) $e^x\tan x$ (120) $e^x(\log x)^2$

(121) $\frac{e^x}{(x+4)^4}$ (122) $\frac{e^x}{x+4}$ (123) $x\cdot e^{\sin^{-1}x}$ (124) $\frac{x}{\log x}$

(125) $x\cdot\operatorname{cosec}(\log x)$ (126) $\sqrt{5}\left[\frac{x}{2}\sqrt{x^2+\frac{3}{5}}+\frac{3}{10}\log\left|x+\sqrt{x^2+\frac{3}{5}}\right|\right]$

(127) $\sin x\sqrt{4\sin^2x-9}-\frac{9}{4}\log\left|2\sin x+\sqrt{4\sin^2x-9}\right|$

(128) $\frac{1}{\log 2}\left[\frac{2^x}{2}\sqrt{4^x+4}+2\log\left|2^x+\sqrt{4^x+4}\right|\right]$

(129) $\frac{4x-3}{8}\sqrt{4+3x-2x^2}+\frac{41}{16\sqrt{2}}\sin^{-1}\left(\frac{4x-3}{\sqrt{41}}\right)$

(130) $\sqrt{2}\left[\frac{4x+3}{8}\sqrt{x^2+\frac{3x}{2}+2}+\frac{23}{16\sqrt{2}}\log\left|x+\frac{3}{4}+\sqrt{x^2+\frac{3x}{2}+2}\right|\right.$

(131) $\frac{\tan x-1}{2}\sqrt{7+2\tan x-\tan^2x}+4\sin^{-1}\left(\frac{\tan x-1}{2\sqrt{2}}\right)$

(132) $\frac{1}{2}\left[-\operatorname{cosec}x\cot x+\log|\operatorname{cosec}x-\cot x|\right]$

(133) $\frac{x^2}{2}\cos^{-1}x-\frac{x}{4}\sqrt{1-x^2}+\frac{1}{4}\sin^{-1}x$ (134) $\frac{e^{5x}}{169}(5\cos 12x+12\sin 12x)$

$$\therefore I = \tan^{-1}x \int x^3\,dx - \int\left[\frac{d}{dx}\tan^{-1}x \int x^3\,dx\right]dx$$

$$= \tan^{-1}x \cdot \frac{x^4}{4} - \int\left[\frac{1}{1+x^2}\cdot\frac{x^4}{4}\right]dx$$

$$= \frac{1}{4}\left[x^4\tan^{-1}x - \int \frac{x^4}{x^2+1}\,dx\right]$$

$$= \frac{1}{4}\left[x^4\tan^{-1}x - \int \frac{x^4-1+1}{x^2+1}\,dx\right]$$

$$= \frac{1}{4}\left[x^4\tan^{-1}x - \int \frac{(x^2-1)(x^2+1)+1}{x^2+1}\,dx\right]$$

splitting

$$= \frac{1}{4}\left[x^4\tan^{-1}x - \int\left\{\frac{(x^2-1)\cancel{(x^2+1)}}{\cancel{x^2+1}} + \frac{1}{x^2+1}\right\}dx\right]$$

$$= \frac{1}{4}\left[x^4\tan^{-1}x - \int\left\{x^2-1+\frac{1}{x^2+1}\right\}dx\right]$$

$$\boxed{\therefore I = \frac{1}{4}\left[x^4\tan^{-1}x - \frac{x^3}{3} + x - \tan^{-1}x\right] + c}$$

$\boxed{Q-48}$ $I = \int e^{\cos x}\sin 2x\,dx$ ← e^u where $u = \cos x$ [Type-4, S.T.(A) Prob.(12)]

$$\therefore I = \int 2\cos x\, e^{\cos x}\sin x\,dx$$

Let $\boxed{\cos x = t}$

Diff. w.r.t. x

$$\therefore -\sin x = \frac{dt}{dx} \qquad \boxed{\therefore \sin x\,dx = -dt}$$

$$\therefore I = \int 2t\cdot e^t(-dt)$$

$$\therefore I = -2\int \underset{(u)}{t}\cdot \underset{(v)}{e^t}\,dt \qquad (\text{ILATE})$$

Integrating by parts

$$\therefore I = -2\left\{t\int e^t\,dt - \int\left[\frac{d}{dt}t\int e^t\,dt\right]dt\right\}$$

$$= -2\left\{t\cdot e^t - \int[(1)(e^t)]\,dt\right\}$$

$$= -2\left\{t\cdot e^t - \int e^t\,dt\right\}$$

$\therefore I = -2\{t \cdot e^t - e^t\} + c$

$\therefore I = -2e^t(t-1) + c$

Backsubstituting

$$\boxed{\therefore I = -2e^{\cos x}(\cos x - 1) + c}$$

Q-49 $I = \int \tan^{-1}x \, dx$ [Type-4, S.T.(B) Prob.(2)]

$\therefore I = \int \underset{(u)}{\tan^{-1}x} \cdot \underset{(v)}{1\,dx}$ (ILATE)

Integrating by parts

$\therefore I = \tan^{-1}x \int 1\,dx - \int \left[\frac{d}{dx}\tan^{-1}x \int 1\,dx\right] dx$

$= \tan^{-1}x \cdot x - \int \left[\frac{1}{1+x^2} \cdot x\right] dx$

$= x \cdot \tan^{-1}x - \frac{1}{2}\int \frac{2x}{1+x^2}\,dx$ $\quad \frac{f'(x)}{f(x)}$

$$\boxed{\therefore I = x \cdot \tan^{-1}x - \frac{1}{2}\log|1+x^2| + c}$$ $\left\{\because \int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + c\right\}$

Q-50 $I = \int (\log x)^2\,dx$ [Type-4, S.T.(B) Prob.(4)]

$\therefore I = \int \underset{(u)}{(\log x)^2} \cdot \underset{(v)}{1\,dx}$ (ILATE)

Integrating by parts

u^2 where $u = \log x$

$\therefore I = (\log x)^2 \int 1\,dx - \int \left[\frac{d}{dx}(\log x)^2 \int 1\,dx\right] dx$

$2u\frac{du}{dx}$

$= (\log x)^2 \cdot x - \int \left[\left\{2\log x \frac{d}{dx}\log x\right\} \cdot x\right] dx$

$= x(\log x)^2 - \int \left[2 \cdot \log x \cdot \frac{1}{\cancel{x}} \cdot \cancel{x}\right] dx$

$= x(\log x)^2 - 2\int \underset{(u)}{\log x} \cdot \underset{(v)}{1\,dx}$ (ILATE)

Integrating by parts

$$\therefore I = x(\log x)^2 - 2\left\{\log x \int 1\,dx - \int\left[\frac{d}{dx}\log x \int 1\,dx\right]dx\right\}$$

$$= x(\log x)^2 - 2\left\{\log x \cdot x - \int\left[\frac{1}{\cancel{x}}\cdot\cancel{x}\right]dx\right\}$$

$$= x(\log x)^2 - 2\left\{x\log x - \int 1\,dx\right\}$$

$$= x(\log x)^2 - 2\left\{x\log x - x\right\} + c$$

$$\boxed{\therefore I = x\left[(\log x)^2 - 2\log x + 2\right] + c}$$

Q-51 $I = \int \log(x^2+1)$ [Type-4, S.T. (B) Prob. (6)]

$$\therefore I = \int \underset{(u)}{\log(x^2+1)} \cdot \underset{(v)}{1}\,dx \qquad \text{(ILATE)}$$

Integrating by parts

$$\therefore I = \log(x^2+1)\int 1\,dx - \int\left[\frac{d}{dx}\log(x^2+1)\int 1\,dx\right]dx$$

$$= \log(x^2+1)\cdot x - \int\left[\left\{\frac{1}{x^2+1}\,\frac{d(x^2+1)}{dx}\right\}\cdot x\right]dx$$

$$= x\cdot\log(x^2+1) - \int\left[\frac{1}{x^2+1}(2x)\cdot x\right]dx$$

$$= x\log(x^2+1) - 2\int\frac{x^2}{x^2+1}\,dx \qquad \text{"Adjustment"}$$

$$= x\log(x^2+1) - 2\int\frac{(x^2+1)-1}{x^2+1}\,dx$$

$$= x\log(x^2+1) - 2\int\left[\frac{\cancel{x^2+1}}{\cancel{x^2+1}} - \frac{1}{x^2+1}\right]dx$$

$$\boxed{\therefore I = x\log(x^2+1) - 2\left[x - \tan^{-1}x\right] + c}$$

Q-52 $I = \int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx$ [Type-4, S.T. (C) Prob. (2)]

Spliting

$$= \int e^x\left[\frac{1}{1+\cos x} + \frac{\sin x}{1+\cos x}\right]dx$$

$$= \int e^x\left[\frac{1}{2\cos^2\frac{x}{2}} + \frac{\cancel{2}\sin\frac{x}{2}\,\cancel{\cos\frac{x}{2}}}{\cancel{2}\cos^{\cancel{2}}\frac{x}{2}}\right]dx$$

$$\therefore I = \int e^x \left[\tan\frac{x}{2} + \frac{1}{2}\sec^2\frac{x}{2} \right] dx$$

Here $f(x) = \tan\frac{x}{2}$

$$\therefore f'(x) = \sec^2\frac{x}{2}\,\frac{d}{dx}\left(\frac{x}{2}\right) = \sec^2\frac{x}{2}\cdot\frac{1}{2}(1) = \frac{1}{2}\sec^2\frac{x}{2}$$

$$\therefore I = \int e^x \left[f(x) + f'(x) \right] dx$$

$$= e^x \cdot f(x) + c$$

$$\boxed{\therefore I = e^x \cdot \tan\frac{x}{2} + c}$$

Q-53 $I = \int e^x \left[\frac{x+2}{(x+3)^2} \right] dx$ [Type-4, S.T. (C) Prob.(4)]

$$= \int e^x \left[\frac{(x+3) - 1}{(x+3)^2} \right] dx$$

spliting

$$= \int e^x \left[\frac{x+3}{(x+3)^2} - \frac{1}{(x+3)^2} \right] dx$$

$$\therefore I = \int e^x \left[\frac{1}{x+3} + \frac{-1}{(x+3)^2} \right] dx$$

Here $f(x) = \frac{1}{x+3}$ $\therefore f'(x) = -\frac{1}{(x+3)^2}$

$$\therefore I = \int e^x \left[f(x) + f'(x) \right] dx$$

$$= e^x \cdot f(x) + c$$

$$\boxed{\therefore I = \frac{e^x}{x+3} + c}$$

Q-54 $I = \int e^{5x} \left[\frac{5x\log x + 1}{x} \right] dx$ [Type-4, S.T. (C) Prob. (6)]

e^u where $u = 5x$

Let $\boxed{5x = t}$

$$\boxed{\therefore x = \frac{t}{5}}$$

Diff. w.r.t. t

$$\therefore \frac{dx}{dt} = \frac{1}{5}$$

$$\boxed{\therefore dx = \frac{dt}{5}}$$

$\leftarrow \cos u$ where $u = \sqrt[3]{x}$

Q-97 $I = \int \cos \sqrt[3]{x} \, dx$ [H.W.E. Prob. (113)]

Let $\boxed{\sqrt[3]{x} = t}$

Taking cube of both sides

$\boxed{\therefore x = t^3}$

Diff. w.r.t. t

$\therefore \frac{dx}{dt} = 3t^2$ $\boxed{\therefore dx = 3t^2 \, dt}$

$\therefore I = \int \cos t \cdot 3t^2 \, dt$ (ILATE)

$\therefore I = 3 \int \underset{(u)}{t^2} \cdot \underset{(v)}{\cos t} \, dt$

Integrating by parts

$= 3\left\{ t^2 \int \cos t \, dt - \int \left[\frac{d}{dt} t^2 \int \cos t \, dt \right] dt \right\}$

$= 3\left\{ t^2(-\sin t) - \int \left[(2t)(-\sin t) \right] dt \right\}$

$= 3\left\{ -t^2 \sin t + 2 \int \underset{(u)}{t} \cdot \underset{(v)}{\sin t} \, dt \right\}$ (ILATE)

Integrating by parts

$= 3\left\{ -t^2 \sin t + 2 \left\{ t \int \sin t \, dt - \int \left[\frac{d}{dt} t \int \sin t \, dt \right] dt \right\} \right\}$

$= 3\left\{ -t^2 \sin t + 2 \left\{ t(-\cos t) - \int \left[(1)(-\cos t) \right] dt \right\} \right\}$

$= 3\left\{ -t^2 \sin t + 2 \left\{ -t \cos t + \int \cos t \, dt \right\} \right\}$

$\therefore I = 3\left\{ -t^2 \sin t - 2t \cos t + 2 \sin t \right\} + c$

Backsubstituting

$$\boxed{\therefore I = 3\left\{ -x^{2/3} \sin \sqrt[3]{x} - 2\sqrt[3]{x} \cos \sqrt[3]{x} + 2 \sin \sqrt[3]{x} \right\} + c}$$

Q-98 $I = \int \log\left(x + \sqrt{x^2 + a^2}\right) dx$ [H.W.E. Prob. (117)]

$\therefore I = \int \underset{(u)}{\log\left(x + \sqrt{x^2 + a^2}\right)} \cdot \underset{(v)}{1} \, dx$ (ILATE)

Integrating by parts

$$\therefore I = \log(x+\sqrt{x^2+a^2}) \int 1\,dx - \int\left[\frac{d}{dx}\log(x+\sqrt{x^2+a^2}) \int 1\,dx\right]dx$$

$$= \log(x+\sqrt{x^2+a^2}) \cdot x - \int\left[\frac{1}{x+\sqrt{x^2+a^2}} \frac{d}{dx}(x+\sqrt{x^2+a^2}) \int 1\,dx\right]dx$$

$$= x\log(x+\sqrt{x^2+a^2}) - \int\left[\frac{1}{x+\sqrt{x^2+a^2}}\left\{1+\frac{1}{2\sqrt{x^2+a^2}}\frac{d}{dx}(x^2+a^2)\right\}\cdot x\right]dx$$

$$= x\log(x+\sqrt{x^2+a^2}) - \int\left[\frac{1}{x+\sqrt{x^2+a^2}}\left\{1+\frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+a^2}}\right\}\cdot x\right]dx$$

$$= x\log(x+\sqrt{x^2+a^2}) - \int\left[\frac{1}{\cancel{x+\sqrt{x^2+a^2}}}\left\{\frac{\cancel{\sqrt{x^2+a^2}+x}}{\sqrt{x^2+a^2}}\right\}\cdot x\right]dx$$

$$= x\log(x+\sqrt{x^2+a^2}) - \int \frac{x}{\sqrt{x^2+a^2}}\,dx$$

$$= x\log(x+\sqrt{x^2+a^2}) - \frac{1}{2}\int \frac{2x}{\sqrt{x^2+a^2}}\,dx \qquad \frac{f'(x)}{\sqrt{f(x)}}$$

$$= x\log(x+\sqrt{x^2+a^2}) - \frac{1}{\cancel{2}}(\cancel{2}\sqrt{x^2+a^2}) + c \qquad \left\{\because \int\frac{f'(x)}{\sqrt{f(x)}}dx = 2\sqrt{f(x)}+c\right\}$$

$$\boxed{\therefore I = x\log(x+\sqrt{x^2+a^2}) - \sqrt{x^2+a^2} + c}$$

Q-99 $I = \int e^{\sin^{-1}x}\left[\frac{x+\sqrt{1-x^2}}{\sqrt{1-x^2}}\right]dx$ $\leftarrow e^u$ where $u = \sin^{-1}x$ [H.W.E. Prob. (123)]

Let $\boxed{\sin^{-1}x = t}$ $\boxed{\therefore x = \sin t}$

Diff w.r.t. x

$\therefore \frac{1}{\sqrt{1-x^2}} = \frac{dt}{dx}$ $\boxed{\therefore \frac{dx}{\sqrt{1-x^2}} = dt}$

Consider $\sqrt{1-x^2} = \sqrt{1-\sin^2 t} = \sqrt{\cos^2 t} = \cos t$

$$\therefore I = \int e^t(\sin t + \cos t)\,dt$$

Here $f(t) = \sin t$ $\therefore f'(t) = \cos t$

$$\therefore I = \int e^t[f(t)+f'(t)]\,dt$$

$\therefore I = e^t \cdot f(t) + c$ $\therefore I = e^t \sin t + c$