

SYSTEM OF PARTICLES AND ROTATIONAL MOTION

Single Correct Answer Type

1. A constant torque acting on a uniform circular wheel changes its angular momentum from A_0 to $4A_0$ in 4s. The magnitude of this torque is

a) $\frac{3A_0}{4}$	b) A_0
c) $4A_0$	d) $12A_0$
2. If the moment of inertia of a disc about an axis tangential and parallel to its surface be I , then what will be the moment of inertia about the axis tangential but perpendicular to the surface?

a) $\frac{6}{5}I$	b) $\frac{3}{4}I$
c) $\frac{3}{2}I$	d) $\frac{5}{4}I$
3. A small block of mass M moves with velocity 5 ms^{-1} towards another block of same mass M placed at a distance of 2 m on a rough horizontal surface. Coefficient of friction between the block and ground is 0.25. Collision between the two blocks is elastic, the separation between the blocks, when both of them come to rest, is ($g=10 \text{ ms}^{-2}$)

a) 3 m	b) 4 m
c) 2 m	d) 1.5 m
4. The moment of inertia of a solid sphere of mass M and radius R about the tangent on its surface is

a) $\frac{7}{5}MR^2$	b) $\frac{4}{5}MR^2$
c) $\frac{2}{5}MR^2$	d) $\frac{1}{2}MR^2$
5. A body of mass M at rest explodes into three pieces, two of which of mass $M/4$ each are thrown off in mutually perpendicular directions with speeds of 3 ms^{-1} and 4 ms^{-1} respectively. Then the third piece will be thrown off with a speed of

a) 1.5 ms^{-1}	b) 2 ms^{-1}
c) 2.5 ms^{-1}	d) 3.0 ms^{-1}
6. A 2 kg mass is rotating on a circular path of radius 0.8 m with angular velocity of 44 rad/sec. If radius of path becomes 1 m. Then the value of angular velocity will be

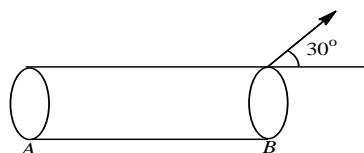
a) 28.16 rad/sec	b) 35.16 rad/sec
c) 19.28 rad/sec	d) 8.12 rad/sec
7. A particle of mass $m = 5$ units is moving with a uniform speed $v = 3\sqrt{2}$ m in the XOY plane along the line $Y = X + 4$. The magnitude of the angular momentum about origin is

a) Zero	b) 60 unit
c) 7.5 unit	d) $40\sqrt{2}$ unit
8. A ladder rests against a frictionless vertical wall, with its upper end 6m above the ground and the lower end 4m away from the wall. The weight of the ladder is 500 N and its C.G. at 1/3rd distance from the lower end. Wall's reaction will be, (in newton)

a) 111	b) 333
c) 222	d) 129
9. Two bodies of masses 2 kg and 4 kg are moving with velocities 20 ms^{-1} and 10 ms^{-1} towards each other due to mutual gravitation attraction. What is the velocity of their centre of mass?

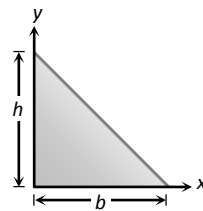
a) 5 ms^{-1}	b) 6 ms^{-1}
c) 8 ms^{-1}	d) Zero

72. A straight rod of length L has one of its ends at the origin and the other at $x = L$. If the mass per unit length of the rod is given by Ax is constant, where is its mass centre?
 a) $L/3$ b) $L/2$
 c) $2L/3$ d) $3L/4$
73. If the angular momentum of a rotating body about a fixed axis is increased by 10%. Its kinetic energy will be increased by
 a) 10% b) 20%
 c) 21% d) 5%
74. Angular momentum L of body with mass moment of inertia I and angular velocity ω rad/sec is equal to
 a) $\frac{I}{\omega}$ b) $I\omega^2$
 c) $I\omega$ d) None of these
75. Two blocks of masses m_1 and m_2 are connected by a massless spring and placed at smooth surface. The spring initially stretched and released. then
 a) The momentum of each particle remains constant separately
 b) The magnitude of momentum of both bodies are same to each other
 c) The mechanical energy of system remains constant
 d) Both (b) and (c) are correct.
76. A small object of mass m is attached to a light string which passes through a hollow tube. The tube is held by one hand and the string by the other. The object is set into rotation in a circle of radius R and velocity v . The string is then pulled down, shortening the radius of path of r . What is conserved
 a) Angular momentum b) Linear momentum
 c) Kinetic energy d) None of the above
77. A solid cylinder rolls down an inclined plane of height 3 m and reaches the bottom of plane with angular velocity of $2\sqrt{2}\text{ rad}\cdot\text{s}^{-1}$. The radius of cylinder must be [Take $g = 10\text{ms}^{-2}$]
 a) 5 cm b) 0.5 cm
 c) $\sqrt{10}\text{ cm}$ d) $\sqrt{5}\text{ m}$
78. The moment of inertia of a circular ring of radius r and mass M about diameter is
 a) Mr^2 b) $\frac{1}{2}Mr^2$
 c) $\frac{3}{2}Mr^2$ d) $\frac{1}{4}Mr^2$
79. A particle of mass m moving with a velocity $(3\hat{i} + 2\hat{j})\text{ms}^{-1}$ collides with a stationary body mass M and finally moves with a velocity $(-2\hat{i} + \hat{j})\text{ms}^{-1}$. If $\frac{m}{M} = \frac{1}{13}$, then
 a) The impulse received by each is, $m(5\hat{i} + \hat{j})$
 b) The velocity of the M is $\frac{1}{13}(5\hat{i} + \hat{j})$
 c) The coefficient of restitution is $\frac{11}{7}$
 d) All the above are correct
80. The instantaneous velocity of a point B of the given rod of length 0.5 m is 3 ms^{-1} in the represented direction. The angular velocity of the rod for minimum velocity of end A is



- a) 1.5 rads^{-1} b) 5.2 rads^{-1}
 c) 2.5 rads^{-1} d) None of these

89. A solid sphere is given a kinetic energy E . What fraction of kinetic energy is associated with rotation?
 a) $3/7$ b) $5/7$
 c) $1/2$ d) $2/7$
90. Three rods each of length L and mass M are placed along X, Y and Z axes in such a way that one end of each rod is at the origin. The moment of inertia of the system about Z -axis is
 a) $\frac{ML^2}{3}$ b) $\frac{2ML^2}{3}$
 c) $\frac{3ML^2}{2}$ d) $\frac{2ML^2}{12}$
91. A diatomic molecule is formed by two atoms which may be treated as mass points m_1 and m_2 , joined by a massless rod of length r . Then the moment of inertia of the molecule about an axis passing through the centre of mass and perpendicular to rod is
 a) zero b) $(m_1 + m_2)r^2$
 c) $\left(\frac{m_1+m_2}{m_1m_2}\right)r^2$ d) $\left(\frac{m_1m_2}{m_1+m_2}\right)r^2$
92. Two rings have their moments of inertia in the ratio 2:1 and their diameters are in the ratio 2:1. The ratio of their masses will be
 a) 2 : 1 b) 1 : 2
 c) 1 : 4 d) 1 : 1
93. A 10 kg body hangs at rest from a rope wrapped around a cylinder 0.2 m in diameter. The torque applied about the horizontal axis of the cylinder is
 a) $98 \text{ N}\cdot\text{m}$ b) $19.6 \text{ N}\cdot\text{m}$
 c) $196 \text{ N}\cdot\text{m}$ d) $9.8 \text{ N}\cdot\text{m}$
94. A rectangular block has a square base measuring $a \times a$, and its height is h . It moves on a horizontal surface in a direction perpendicular to one of its edges. The coefficient of friction is μ . It will topple if
 a) $\mu > h/a$ b) $\mu > a/h$
 c) $\mu > \frac{2a}{h}$ d) $\mu > \frac{a}{2h}$
95. A 2 kg body and a 3 kg body are moving along the x -axis. At a particular instant the 2 kg body has a velocity of 3 ms^{-1} and the 3 kg body has the velocity of 2 ms^{-1} . The velocity of the centre of mass at that instant is
 a) 5 ms^{-1} b) 1 ms^{-1}
 c) 0 d) None of these
96. The centre of mass of triangle shown in figure has coordinates

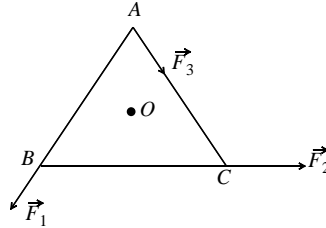


- a) $x = \frac{h}{2}, y = \frac{b}{2}$ b) $x = \frac{b}{2}, y = \frac{h}{2}$
 c) $x = \frac{b}{3}, y = \frac{h}{3}$ d) $x = \frac{h}{3}, y = \frac{b}{3}$
97. A circular turn table has a block of ice placed at its centre. The system rotates with an angular speed ω about an axis passing through the centre of the table. If the ice melts on its own without any evaporation, the speed of rotation of the system
 a) Becomes zero
 b) Remains constant at the same value ω
 c) Increases to a value greater than ω
 d) Decreases to a value less than ω

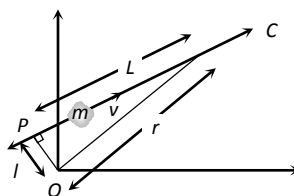


98. Two masses m_1 and m_2 ($m_1 > m_2$) are connected by massless flexible and inextensible string passed over massless and frictionless pulley. The acceleration of centre of mass is
- a) $\left(\frac{m_1-m_2}{m_1+m_2}\right)^2 g$ b) $\frac{m_1-m_2}{m_1+m_2} g$
c) $\frac{m_1+m_2}{m_1-m_2} g$ d) Zero
99. Which relation is not correct of the following
- a) Torque= Moment of inertia \times angular acceleration
b) Torque=Dipole moment \times magnetic induction
c) Moment of inertia = Torque/angular acceleration
d) Liner momentum = Moment of inertia \times angular velocity
100. A particle moves in the x - y plane under the action of a force F such that the value of its linear momentum \vec{P} at any time t is $p_x = 2 \cos t, p_y = 2 \sin t$
The angel θ between \vec{F} and \vec{P} at a given time t will be
- a) 90° b) 0°
c) 180° d) 30°
101. A wheel has angular acceleration of 3.0 rad/sec^2 and an initial angular speed of 2.00 rad/sec . In a time of 2 sec it has rotated through an angle (In radian) of
- a) 6 b) 10
c) 12 d) 4
102. A torque of 50 Nm acting on a wheel at rest rotates it through 200 radians in 5 sec . Calculate the angular acceleration produced
- a) 8 rad sec^{-2} b) 4 rad sec^{-2}
c) 16 rad sec^{-2} d) 12 rad sec^{-2}
103. Two discs of moment of inertia I_1 and I_2 and angular speeds ω_1 and ω_2 are rotating along collinear axes passing through their centre of mass and perpendicular to their plane. If the two are made to rotate combindly along the same axis the rotational KE of system will be
- a) $\frac{I_1\omega_1+I_2\omega_2}{2(I_1+I_2)}$ b) $\frac{(I_1+I_2)(\omega_1+\omega_2)^2}{2}$
c) $\frac{(I_1\omega_1+I_2\omega_2)^2}{2(I_1+I_2)}$ d) None of these
104. Four particles each of mass m are placed at the corners of a square of side length l . The radius of gyration of the system about an axis perpendicular to the square and passing through its centre is
- a) $\frac{l}{\sqrt{2}}$ b) $\frac{l}{2}$
c) l d) $(\sqrt{2})l$
105. If the angular momentum of any rotating body increases by 200% , then the increase in its kinetic energy
- a) 400% b) 800%
c) 200% d) 100%
106. A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same which one of the following will not be affected?
- a) Moment of inertia b) Angular momentum
c) Angular velocity d) Rotational kinetic energy
107. A disc is rolling (without slipping) on a horizontal surface C is its centre and Q and P are two points equidistant from C . Let v_P, v_Q and v_C be the magnitude of velocities of pints P, Q and C respectively, then

117. ABC is an equilateral triangle with O as its centre. \vec{F}_1, \vec{F}_2 and \vec{F}_3 represent three forces acting along the sides AB, BC and AC respectively. If the total torque about O is zero then the magnitude of \vec{F}_3 is



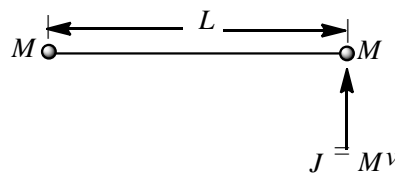
- | | |
|------------------------|-------------------|
| a) $F_1 + F_2$ | b) $F_1 - F_2$ |
| c) $\frac{F_1+F_2}{2}$ | d) $2(F_1 + F_2)$ |
118. If rotational kinetic energy is 50% of translational kinetic energy, then the body is
- | | |
|------------------|-----------------|
| a) Ring | b) Cylinder |
| c) Hollow sphere | d) Solid sphere |
119. The moment of inertia of thin circular disc about an axis passing through its centre and perpendicular to its plane is I . Then, the moment of inertia of the disc about an axis parallel to its diameter and touching the edge of the rim is
- | | |
|-------------------|-------------------|
| a) I | b) $2I$ |
| c) $\frac{3}{2}I$ | d) $\frac{5}{2}I$ |
120. Three identical thin rods each of length l and mass M are joined together to form a letter H . What is the moment of inertia of the system about one of the sides of H ?
- | | |
|-----------------------|-----------------------|
| a) $M \frac{l^2}{4}$ | b) $M \frac{l^2}{3}$ |
| c) $2 \frac{Ml^2}{3}$ | d) $4 \frac{Ml^2}{3}$ |
121. Two bodies A and B have masses M and m respectively, where $M > m$ and they are at a distance d apart. Equal force is applied to them so that they approach each other. The position where they hit each other is
- | | |
|---------------------------------------|----------------------|
| a) Nearer to B | b) Nearer to A |
| c) At equal distance from A and B | d) Cannot be decided |
122. A thin metal disc of radius 0.25 m and mass 2 kg starts from rest and rolls down an inclined plane. If its rotational kinetic energy is 4 J at the foot of the inclined plane, then its linear velocity at the same point is
- | | |
|--------------------------|--------------------------------|
| a) 1.2 ms^{-1} | b) $2\sqrt{2} \text{ ms}^{-1}$ |
| c) 20 ms^{-1} | d) 2 ms^{-1} |
123. If a hollow cylinder and a solid cylinder are allowed to roll down an inclined plane, which will take more time to reach the bottom
- | | |
|--------------------|------------------------------|
| a) Hollow cylinder | b) Solid cylinder |
| c) Same for both | d) One whose density is more |
124. A particle of mass m moves along line PC with velocity v as shown. What is the angular momentum of the particle about O



- | | |
|----------|----------|
| a) mvL | b) mvL |
| c) mvr | d) Zero |

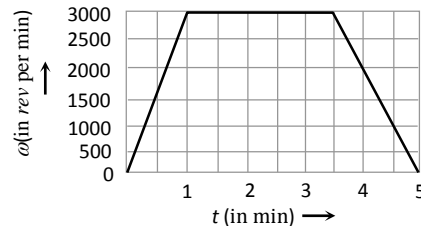


125. A bomb of mass 9 kg explodes into two pieces of masses 3 kg and 6 kg. The velocity of mass 3 kg is 16 ms^{-1} . The kinetic energy of mass 6 kg in joule is
 a) 96
 b) 384
 c) 192
 d) 768
126. A body of mass M moving with velocity $v \text{ ms}^{-1}$ suddenly breaks into two pieces. One part having mass $M/4$ remains stationary. The velocity of the other part will be
 a) v
 b) $2v$
 c) $\frac{3v}{4}$
 d) $\frac{4v}{3}$
127. Consider a body, shown in figure, consisting of two identical balls, each of mass M connected by a light rigid rod. If an impulse $J = Mv$ is imparted to the body at one of its ends, what would be its angular velocity?



- a) v/L
 b) $2v/L$
 c) $v/3L$
 d) $v/4L$
128. The angle turned by a body undergoing circular motion depends on time as $\theta = \theta_0 + \theta_1 t + \theta_2 t^2$. Then the angular acceleration of the body is
 a) θ_1
 b) θ_2
 c) $2\theta_1$
 d) $2\theta_2$
129. Circular hole of radius 1 cm is cut off from a disc of radius 6 cm. The centre of hole is 3 cm from the centre of the disc. The position of centre of mass of the remaining disc from the centre of disc is
 a) $-\frac{3}{35} \text{ cm}$
 b) $\frac{3}{35} \text{ cm}$
 c) $\frac{3}{10} \text{ cm}$
 d) None of these
130. Two spherical bodies of masses M and $5M$ in free space with initial separation between their centres equal to $12R$. If they attract each other due to gravitational force only, then the distance covered by the smaller body just before collision is
 a) $2.5 R$
 b) $4.5 R$
 c) $7.5 R$
 d) $1.5 R$
131. Two perfectly elastic objects A and B of identical mass are moving with velocities 15 ms^{-1} and 10 ms^{-1} respectively, collide along the direction of line joining them. Their velocities after collision are respectively
 a) $10 \text{ ms}^{-1}, 15 \text{ ms}^{-1}$
 b) $20 \text{ ms}^{-1}, 5 \text{ ms}^{-1}$
 c) $0 \text{ ms}^{-1}, 25 \text{ ms}^{-1}$
 d) $5 \text{ ms}^{-1}, 20 \text{ ms}^{-1}$
132. Consider the following two statements
 I. Linear momentum of a system of particles is zero.
 II. Kinetic energy of a system of particles is zero. Then
 a) I implies II and II implies I
 b) I does not imply II and II does not imply I
 c) I implies II but II does not imply I
 d) I does not imply II but II implies I
133. A wheel is rotating at 900 r.p.m. about its axis. When the power is cut-off, it comes to rest in 1 minute. The angular retardation in radian/s^2 is
 a) $\pi/2$
 b) $\pi/4$
 c) $\pi/6$
 d) $\pi/8$

143. As a part of a maintenance inspection the compressor of a jet engine is made to spin according to the graph as shown. The number of revolutions made by the compressor during the test is



- a) 9000
 b) 16570
 c) 12750
 d) 11250
144. The moment of inertia of a solid cylinder of mass M and radius R about a line parallel to the axis of the cylinder and lying on the surface of the cylinder is
- a) $\frac{2}{5}MR^2$
 b) $\frac{3}{5}MR^2$
 c) $\frac{3}{2}MR^2$
 d) $\frac{5}{2}MR^2$
145. A disc is rotating with an angular speed of ω . If a child sits on it, which of the following is conserved
- a) Kinetic energy
 b) Potential energy
 c) Linear momentum
 d) Angular momentum
146. Identify the correct statement for the rotational motion of a rigid body
- a) Individual particles of the body do not undergo accelerated motion
 b) The centre of mass of the body remains unchanged
 c) The centre of mass of the body moves uniformly in a circular path
 d) Individual particles and centre of mass of the body undergo an accelerated motion
147. A spacecraft of mass M is moving with velocity v in free space when it explodes and breaks in two. After the explosion, a mass m of the spacecraft is left stationary. What is the velocity of other part?
- a) $\frac{Mv}{(M-m)}$
 b) $\frac{mv}{(M+m)}$
 c) $\frac{mv}{(M-m)}$
 d) $\frac{(M+m)v}{M}$
148. There is a uniform circular disc of radius R and a concentric disc of radius r (where $r < R$) is cut off from it. The distance of the new position of centre of mass of hollow disc from the centre of disc is
- a) $\frac{R-r}{2}$
 b) $R-r$
 c) Zero
 d) $\sqrt{R^2 - r^2}$
149. Four particles of mass 1 kg, 2 kg, 3 kg and 4 kg are placed at the corners A, B, C and D respectively of a square $ABCD$ of edge X -axis and edge AD is taken along Y -axis, the co-ordinates of centre of mass in SI is
- a) (1,1)
 b) (5,7)
 c) (0.5,0.7)
 d) None of these
150. The moment of inertia of uniform rectangular plate about an axis passing through its centre and parallel to its length l is (b = breadth of rectangular plate)
- a) $\frac{Mb^2}{4}$
 b) $\frac{Mb^3}{6}$
 c) $\frac{Mb^3}{12}$
 d) $\frac{Mb^2}{12}$
151. A spring pong ball of mass m is floating in air by a jet of water emerging out of a nozzle. If the water strikes the ping pong ball with a speed v and just after collision water falls dead, the rate of flow of water in the nozzle is equal to
- a) $\frac{2mg}{v}$
 b) $\frac{m}{g}$
 c) $\frac{mg}{v}$
 d) $\frac{2m}{vg}$

152. Angular momentum of a body is defined as the product of
- a) Mass and angular velocity b) Centripetal force and radius
 c) Linear velocity and angular velocity d) Moment of inertia and angular velocity
153. A solid sphere, a hollow sphere and a ring are released from top of an inclined plane (frictionless) so that they slide down the plane. Then maximum acceleration down the plane is for (no rolling)
- a) Solid sphere b) Hollow sphere
 c) Ring d) All same
154. A machine gun fires 120 shoots per minute. If the mass of each bullet is 10 g and the muzzle velocity is 800 ms^{-1} , the average recoil force on the machine gun is
- a) 120 N b) 8 N
 c) 16 N d) 12 N
155. For spheres each of mass M and radius R are placed with their centers on the four corners A, B, C and D of a square of side b . The spheres A and B are hollow and C and D are solids. The moment of inertia of the system about side AD of square is
- a) $\frac{8}{3}MR^2 + 2Mb^2$ b) $\frac{8}{5}MR^2 + 2Mb^2$
 c) $\frac{32}{15}MR^2 + 2Mb^2$ d) $32MR^2 + 4Mb^2$
156. Let \mathbf{F} be the force acting on a particle having position vector \mathbf{r} and $\boldsymbol{\tau}$ be the torque of this force about the origin. Then
- a) $\mathbf{r} \cdot \boldsymbol{\tau} = 0$ and $\mathbf{F} \cdot \boldsymbol{\tau} \neq 0$ b) $\mathbf{r} \cdot \boldsymbol{\tau} \neq 0$ and $\mathbf{F} \cdot \boldsymbol{\tau} = 0$
 c) $\mathbf{r} \cdot \boldsymbol{\tau} \neq 0$ and $\mathbf{F} \cdot \boldsymbol{\tau} \neq 0$ d) $\mathbf{r} \cdot \boldsymbol{\tau} = 0$ and $\mathbf{F} \cdot \boldsymbol{\tau} = 0$
157. A tennis ball bounces down flight of stairs striking each step in turn and rebounding to the height of the step above. The coefficient of restitution has a value
- a) $\frac{1}{2}$ b) 1
 c) $1/\sqrt{2}$ d) $1/2\sqrt{2}$
158. The center of mass of three particles of masses 1 kg, 2 kg and 3 kg at (3, 3, 3) with reference to a fixed coordinate system. Where should a fourth particle of mass 4 kg should be placed, so that the center of mass of the system of all particles shifts to a point (1, 1, 1)?
- a) (-1, -1, -1) b) (-2, -2, -2)
 c) (2, 2, 2) d) (1, 1, 1)
159. A thin circular ring of mass m and radius R is rotating about its axis with a constant angular velocity ω . Two objects each of mass M are attached gently to the opposite ends of a diameter of the ring the ring. The ring now rotates with an angular velocity ω' is equal to
- a) $\frac{\omega(m+2M)}{m}$ b) $\frac{\omega(m-2M)}{(m+2M)}$
 c) $\frac{\omega m}{(m+M)}$ d) $\frac{\omega m}{(m+2M)}$
160. A mass m hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass m and radius R . Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass m , if the string does not slip on the pulley, is
- a) g b) $\frac{2}{3}g$
 c) $\frac{g}{3}$ d) $\frac{3}{2}g$
161. Choose the correct statement about the centre of mass (CM) of a system of two particles
- a) The CM lies on the line joining the two particles midway between them
 b) The CM lies on the line joining them at a point whose distance from each particle is inversely proportional to the mass of that particle
 c) The CM lies on the line joining them at a point whose distance from each particle is proportional to the square of the mass of that particle
 d) The CM is on the line joining them at a point whose distance from each particle is proportional to the mass of that particle

: ANSWER KEY :

1)	A	2)	a	3)	a	4)	a	5)	c	6)	a	7)	b	8)	a
9)	d	10)	c	11)	d	12)	b	13)	c	14)	a	15)	c	16)	b
17)	b	18)	b	19)	b	20)	d	21)	a	22)	a	23)	d	24)	a
25)	b	26)	a	27)	b	28)	c	29)	b	30)	c	31)	b	32)	a
33)	c	34)	d	35)	a	36)	b	37)	d	38)	c	39)	a	40)	b
41)	d	42)	c	43)	d	44)	d	45)	a	46)	a	47)	b	48)	d
49)	b	50)	c	51)	b	52)	d	53)	a	54)	a	55)	d	56)	d
57)	c	58)	c	59)	b	60)	c	61)	a	62)	d	63)	a	64)	d
65)	A	66)	b	67)	c	68)	a	69)	b	70)	b	71)	a	72)	b
73)	c	74)	c	75)	d	76)	a	77)	d	78)	b	79)	d	80)	b
81)	a	82)	a	83)	c	84)	c	85)	c	86)	a	87)	c	88)	a
89)	d	90)	b	91)	d	92)	b	93)	d	94)	b	95)	d	96)	c
97)	d	98)	a	99)	d	100)	a	101)	b	102)	c	103)	c	104)	a
105)	b	106)	b	107)	a	108)	a	109)	d	110)	a	111)	b	112)	d
113)	d	114)	d	115)	c	116)	a	117)	a	118)	b	119)	d	120)	d
121)	b	122)	b	123)	a	124)	b	125)	c	126)	d	127)	a	128)	d
129)	a	130)	c	131)	a	132)	a	133)	a	134)	d	135)	a	136)	b
137)	c	138)	b	139)	a	140)	d	141)	c	142)	c	143)	d	144)	c
145)	d	146)	b	147)	a	148)	c	149)	c	150)	d	151)	c	152)	d
153	d	154)	c	155)	c	156)	d	157)	c	158)	b	159)	d	160)	b
161)	b														

: HINTS AND SOLUTIONS :

1 (a)

$$c = \frac{dL}{dt} = \frac{L_2 - L_1}{\Delta t} = \frac{4A_0 - A_0}{4} = \frac{3A_0}{4}$$

2 (a)

MI of disc about tangent in a plane

$$= \frac{5}{4}MR^2 = I$$

$$\therefore mR^2 = \frac{4}{5}I$$

MI of disc about tangent I to plane $I' = \frac{3}{2}mR^2$

$$\therefore I' = \frac{3}{2}\left(\frac{4}{5}I\right)$$

$$= \frac{6}{5}I$$

3 (a)

Retardation due to friction

$$a = \mu g = (0.25)(10)$$

$$= 2.5 \text{ ms}^{-2}$$

Collision is elastic, i.e. after collision first block comes to rest and the second block acquires the velocity of first block. Or we can understand it in this manner that second block is permanently at rest while only the first block moves. Distance travelled by it will be

$$s = \frac{v^2}{2a} = \frac{(5)^2}{(2)(2.5)} = 5 \text{ m}$$

\therefore Final separation will be $(s - 2) = 3 \text{ m}$

4 (a)

Moment of inertia of the solid sphere of mass M and radius R about its tangent

$$I = I_0 + MR^2$$

(According to theorem of parallel axis)

$$= \frac{2}{5}MR^2 + MR^2 \quad (\because I_0 = \frac{2}{5}MR^2)$$

$$I = \frac{7}{5}MR^2$$

5 (c)

As there is no external force, hence

$$\vec{p} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 = \text{constant}$$

$$\Rightarrow |\vec{p}_3| = |\vec{p}_1 + \vec{p}_2| = \frac{m}{4} \sqrt{(3)^2 + (4)^2} = \frac{5m}{4}$$

[since v_1 and v_2 are mutually perpendicular]

$$\therefore p_3 = \frac{m}{2}v_3 = \frac{5m}{4}$$

$$\Rightarrow v_3 = \frac{5}{2} = 2.5 \text{ ms}^{-1}$$

6 (a)

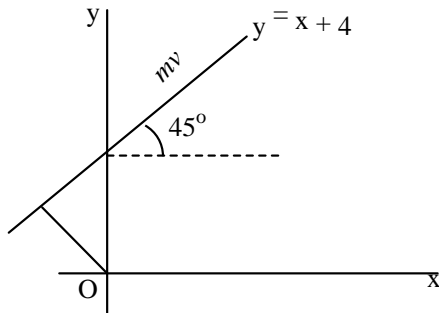
Angular momentum, $L = mr^2\omega = \text{constant}$

$$\frac{\omega_2}{\omega_1} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{0.8}{1}\right)^2 = 0.64 \Rightarrow \omega_2 = 44 \times 0.64 = 28.16 \frac{\text{rad}}{\text{s}}$$

- 7 (b)
Angular momentum about origin

$$|\mathbf{L}| = |\mathbf{r} \times m\mathbf{v}|$$

$$= (4 \cos 45^\circ) \times (5) \times 3\sqrt{2}$$



$$= \frac{a}{\sqrt{2}} \times 5 \times 3\sqrt{2}$$

$$|\mathbf{L}| = 60 \text{ unit}$$

- 8 (a)
111
- 9 (d)

Since net force acting on the system is zero, hence position of centre of mass of the system remains unchanged *ie*, velocity of the centre of mass is zero

- 10 (c)
As is clear from the equation,
 $|(m_1\vec{v}_1 + m_2\vec{v}_2) - (m_1\vec{v}_1 + m_2\vec{v}_2)|$
= change in linear momentum of the two particles
= external force on the system \times time interval
= $[(m_1 + m_2)g] \times (2t_0) = 2(m_1 + m_2)gt_0$

- 11 (d)
At the highest point momentum of particle before explosion $\vec{p} = mv \cos 60^\circ$
 $= m \times 200 \frac{1}{2} = 100m$ horizontally

Now as there is no external force during explosion, hence

$$\vec{p} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3$$

However, since velocities of two fragments, of masses $m/3$ each, are 100 ms^{-1} downward and 100 ms^{-1} upward.

$$\text{hence, } \vec{p}_1 = -\vec{p}_2 \text{ or } \vec{p}_1 + \vec{p}_2 = 0$$

$$\vec{p}_3 = \frac{m}{3} \cdot v_3 = \vec{p} = 100m \text{ horizontally}$$

$$v_3 = 300 \text{ ms}^{-1} \text{ horizontally}$$

- 12 (b)
Since, the acceleration of centre of mass in both the cases is same equal to g . So, the centre of mass of the bodies B and C taken together does not shift compared to that of body A .
As $\vec{\tau}_f \neq 0 \therefore \vec{\tau}_N \neq 0$ and torque by friction and normal reaction will be in opposite direction

- 13 (c)
Linear acceleration for rolling $a = \frac{g \sin \theta}{(1 + K^2/R^2)}$

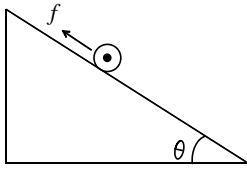
$$\text{For cylinder } \frac{K^2}{R^2} = \frac{1}{2}$$

$$\therefore a_{\text{cylinder}} = \frac{2}{3} g \sin \theta \quad \dots(i)$$

$$\text{For rotation, the torque } fR = I\alpha = (MR^2\alpha)/2$$

(where f = force of friction)

But $R\alpha = a \therefore f = \frac{M}{2}a$



$$\therefore f = \frac{M}{2} \cdot \frac{2}{3}g \sin \theta = \frac{M}{3}g \sin \theta$$

$\mu_s = f/N$ where N is normal reaction, $Mg \cos \theta$

$$\mu_s = \frac{\frac{M}{3}g \sin \theta}{Mg \cos \theta} = \frac{\tan \theta}{3}$$

\therefore For rolling without slipping of a roller down the inclined plane, $\tan \theta \geq 3\mu_s$

14

(a)

Here, effected gravitational acceleration is

$$g' = \frac{mg - qE}{m}$$

$$\therefore R = \frac{v_0^2 \sin 2\alpha}{g'}$$

It means, g' for both particles are same

This is possible when

$$m_1 = m_2 \text{ and } e_1 = e_2$$

15

(c)

Distribution of mass about BC axis is more than that about AB axis, *i. e.* radius of gyration about BC axis is more than that about AB axis

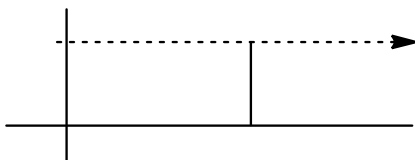
$$i. e. K_{BC} > K_{AB} \therefore I_{BC} > I_{AB} > I_{CA}$$

16

(b)

Angular momentum = (Linear momentum) \times (perpendicular distance to line of motion from the axis)

or angular momentum is moment of momentum. Here, the angle goes on decreasing from 90° but the perpendicular distance to the line of motion remains constant. Therefore, angular momentum is also constant (linear momentum $p = mv$ is constant).



17

(b)

Angular velocity is given by

$$\omega = 600 \text{ rotation/min}$$

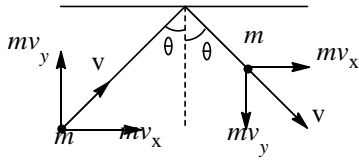
$$= \frac{600 \times 2\pi}{60} \text{ rads}^{-1} = 20\pi \text{ rads}^{-1}$$

Kinetic energy of coin which is due to rotation and translation is

$$\begin{aligned} K &= \frac{1}{2} I\omega^2 + \frac{1}{2}mv^2 \\ &= \frac{1}{2} \times \frac{1}{2}mr^2\omega^2 + \frac{1}{2}m(\omega r)^2 \\ &= \frac{1}{4} \times 4.8 \times (1)^2(20\pi)^2 + \frac{1}{2} \times 4.8 \times (20\pi \times 1)^2 \\ &= 480\pi^2 + 960\pi^2 = 1440\pi^2 \text{ J} \end{aligned}$$

18 (b)

From adjoining figure the component of momentum along x -axis (parallel to the wall of container) remains unchanged even after the collision.



\therefore Impulse = change in momentum of gas molecule along y -axis, *ie*, in a direction normal to the wall = $2mv_y = 2mv \cos \theta$

19 (b)

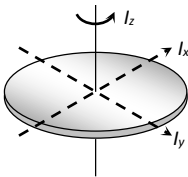
$$\frac{1}{2}MR^2 = MK^2 \Rightarrow K = \frac{R}{\sqrt{2}} = \frac{2.5}{\sqrt{2}} = 1.76 \text{ cm}$$

20 (d)

$$I_z = I_x + I_y$$

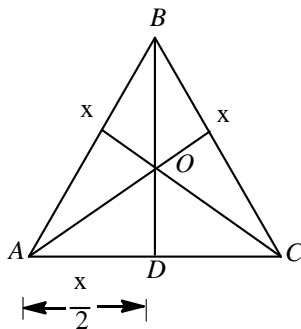
$$200 = I_D + I_D = 2I_D$$

$$\therefore I_D = 100 \text{ g} \times \text{cm}^2$$



21 (a)

The radius of gyration is the distance from the axis of rotation at which if whole mass of the body is supposed to be concentrated. Here, the whole mass of the equilateral triangle acts at point O . So the distance OA is the radius of gyration of this system. Now from triangle ADB



$$x^2 = BD^2 + \left(\frac{x}{2}\right)^2$$

$$\text{or } BD^2 = x^2 - \frac{x^2}{4}$$

$$\text{or } BD^2 = \frac{3x^2}{4}$$

$$\text{or } BD = \sqrt{3} \frac{x}{2}$$

$$\text{Hence, the distance, } OB = \frac{\sqrt{3}x}{2} \times \frac{2}{3}$$

$$\Rightarrow OB = \frac{x}{\sqrt{3}}$$

But, the distances OA , OB and OC are the same.

$$\text{So, } OA = \frac{x}{\sqrt{3}}$$

Hence, the radius of gyration of this system is $\frac{x}{\sqrt{3}}$.

22 (a)
It's always in axial direction

23 (d)
For a ring $K^2 = r^2$ then

$$v^2 = \sqrt{\frac{2gh}{1 + \frac{K^2}{r^2}}}$$

$$\therefore v^2 = \frac{2gh}{2} = gh$$

$$v = \sqrt{gh}$$

24 (a)
As a real velocity of comet is constant, therefore,

$$r_1 v_1 = r_2 v_2 \text{ or } v_2 = \frac{r_1 v_1}{r_2}$$

$$= \frac{6 \times 10^{10} \times 7 \times 10^4}{1.4 \times 10^{12}} = 3 \times 10^3 \text{ms}^{-1}$$

25 (b)
As net horizontal force acting on the system is zero, hence momentum must remain conserved. Hence

$$mu + 0 = 0 + mv_2 \Rightarrow v_2 = \frac{mu}{M}$$

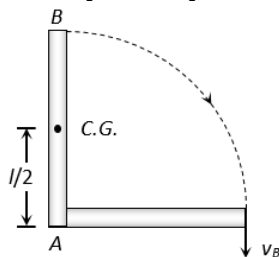
As per definition,

$$e = -\frac{(v_1 - v_2)}{(u_2 - u_1)} = \frac{v_2 - 0}{0 - u} = \frac{v_2}{u} = \frac{\frac{mu}{M}}{u} = \frac{m}{M}$$

26 (a)
Angular momentum of system remains constant

$$I \propto \frac{1}{\omega} \Rightarrow \frac{I_2}{I_1} = \frac{\omega_1}{\omega_2} = \frac{20}{10} \Rightarrow I_2 = 2I_1 = 2I$$

27 (b)
In this process potential energy of the metre stick will be converted into rotational kinetic energy



$$\text{P.E. of meter stick} = mg \left(\frac{l}{2}\right)$$

Because its centre of gravity lies at the middle point of the rod

$$\text{Rotational kinetic energy } E = \frac{1}{2}I\omega^2$$

$$I = \text{M.I. of metre stick about point A} = \frac{ml^2}{3}$$

ω = Angular speed of the rod while striking the ground

v_B = Velocity of end B of metre stick while striking the ground

By the law of conservation of energy,

$$mg \left(\frac{l}{2}\right) = \frac{1}{2}I\omega^2 = \frac{1}{2} \frac{ml^2}{3} \left(\frac{v_B}{l}\right)^2$$

$$\text{By solving we get, } v_B = \sqrt{3gl} = \sqrt{3 \times 10 \times 1} = 5.4 \text{m/s}$$

28

(c)

To keep the centre of mass at the position, velocity of centre of mass is zero, so

$$\frac{m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2}{m_1 + m_2} = 0$$

where \mathbf{v}_1 and \mathbf{v}_2 are velocities of particles 1 and 2 respectively.

$$\Rightarrow m_1 \frac{d\mathbf{r}_1}{dt} + m_2 \frac{d\mathbf{r}_2}{dt} = 0$$

$$[\because \mathbf{v}_1 = \frac{d\mathbf{r}_1}{dt} \text{ and } \mathbf{v}_2 = \frac{d\mathbf{r}_2}{dt}]$$

$$\Rightarrow m d\mathbf{r}_1 + m_2 d\mathbf{r}_2 = 0 \text{ [} d\mathbf{r}_1 \text{ and } d\mathbf{r}_2 \text{ represent the change in displacement of particles]}$$

Let 2nd particle has been displaced by distance x .

$$\Rightarrow m_1(d) + m_2(x) = 0$$

$$\Rightarrow x = -\frac{m_1 d}{m_2}$$

Negative sign shows that both the particles have to move in opposite directions.

So, $\frac{m_1 d}{m_2}$ is the distance moved by 2nd particle to keep centre of mass at the same position.

29

(b)

Torque zero means, α zero

$$\therefore \frac{d^2\theta}{dt^2} = 0 \Rightarrow 12t - 12 = 0$$

$$\therefore t = 1 \text{ second}$$

30

(c)

$$I_s = \frac{2}{5}MR_s^2, I_h = \frac{2}{3}MR_h^2$$

$$\text{As, } I_s = I_h$$

$$\therefore \frac{2}{5}MR_s^2 = \frac{2}{3}MR_h^2$$

$$\therefore \frac{R_s}{R_h} = \frac{\sqrt{5}}{\sqrt{3}}$$

31

(b)

As body is moving on a frictionless surface. Its mechanical energy is conserved. When body climbs up the inclined plane it keeps on rotating with same angular speed, as no friction force is present to provide retarding torque so

$$\frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 \geq \frac{1}{2}I\omega^2 + mgh \Rightarrow v \geq \sqrt{2gh}$$

32

(a)

$$\text{K.E.} = \frac{L^2}{2I}$$

\therefore From angular momentum conservation about centre

$$L \rightarrow \text{constant}$$

$$I = mr^2$$

$$\text{K.E.'} = \frac{L^2}{2(mr'^2)} \quad r' = \frac{r}{2}$$

$$\text{K.E.'} = 4 \text{ K.E.}$$

K.E. is increased by a factor of 4

33

(c)

$$\frac{\text{Rotational kinetic energy}}{\text{Translatory kinetic energy}} = \frac{\frac{1}{2}mv^2 \frac{K^2}{R^2}}{\frac{1}{2}mv^2} = \frac{K^2}{R^2} = \frac{2}{5}$$

34 (d)

From $E = \frac{1}{2} r \omega^2$, we find that when frequency (n) is doubled, $\omega = 2\pi n$ is doubled, ω^2 becomes 4 times.

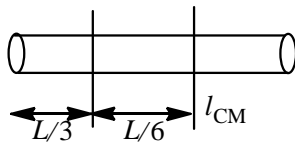
As E reduces to half, I must have been reduced to $\frac{1}{8}$ th. From $L = I\omega$, L becomes $\frac{1}{8} \times 2 = \frac{1}{4}$ times
ie, $0.25 L$

35 (a)

$$\frac{2}{5} MR^2 = \frac{3}{2} Mr^2 \Rightarrow r = \frac{2R}{\sqrt{15}}$$

36 (b)

$$I_{CM} = \frac{ML^2}{12} \quad (\text{about middle point})$$



$$\begin{aligned} \therefore I &= I_{CM} + Mx^2 \\ &= \frac{ML^2}{12} + M\left(\frac{L}{6}\right)^2 \\ I &= \frac{ML^2}{9} \end{aligned}$$

37 (d)

$$I = \frac{2}{5} mr^2 = mk^2$$

$$k^2 = \frac{2}{5} r^2 \Rightarrow k = r\sqrt{0.4}$$

38 (c)

$$I = 2MR^2 = 2 \times 3 \times (1)^2 = 6g - cm^2$$

39 (a)

Given system of two particles will rotate about its centre of mass

$$\text{Initial angular momentum} = MV\left(\frac{L}{2}\right)$$

$$\text{Final angular momentum} = 2I\omega = 2M\left(\frac{L}{2}\right)^2 \omega$$

By the law of conservation of angular momentum

$$MV\left(\frac{L}{2}\right) = 2M\left(\frac{L}{2}\right)^2 \omega \Rightarrow \omega = \frac{V}{L}$$

40 (b)

$$E = K_R = \frac{1}{2} I\omega^2$$

If angular velocities are equal then $E \propto I$

As $I_1 > I_2$ therefore $E_1 > E_2$

41 (d)

When the hands outstretched, moment of inertia increases and angular velocity decreases so that angular momentum remains unchanged

42 (c)

$$\text{Given, } MI = 2.5 \text{ kgm}^{-2}$$

$$W = 40 \text{ rad s}^{-1}$$

$$T = 10 \text{ Nm}$$

$$\text{As } T = I\alpha$$

$$10 = 2.5\alpha$$

$$\alpha = 4 \text{ rad s}^{-2}$$

Now, $\omega = \omega_0 + \alpha t$

$$60 = 40 + 4 \times t$$

$$20 = 4t$$

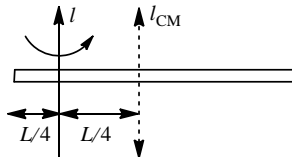
$$t = 5 \text{ s}$$

43 (d)

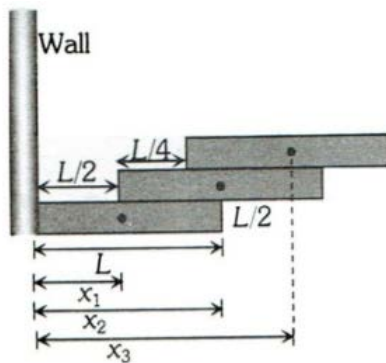
$$I = I_{CM} + Mx^2$$

$$= \frac{ML^2}{12} + M \left[\frac{L}{4} \right]^2$$

$$= \frac{ML^2}{12} + \frac{ML^2}{16} = \frac{7ML^2}{48}$$



44 (d)



From figure, $x_1 = \frac{L}{2}$, $x_2 = \frac{L}{2} + \frac{L}{2} = L$

$$x_3 = \frac{L}{2} + \frac{L}{4} + \frac{L}{2} = \frac{5L}{4}$$

$$\therefore X_{CM} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$= \frac{M \times \frac{L}{2} + M \times L + M \times \frac{5L}{4}}{M + M + M} = \frac{\frac{11}{4} ML}{3M} = \frac{11L}{12}$$

45 (a)

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{0.4 \times 2 + 0.6 \times 7}{0.4 + 0.6} = 5 \text{ m}$$

46 (a)

$$a = \frac{g \sin \theta}{1 + \frac{K^2}{R^2}} = \frac{g \sin \theta}{1 + \frac{5}{2}} = \frac{5}{7} g \sin \theta$$

47 (b)

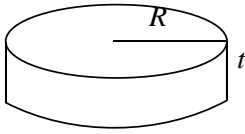
M.I. of a cylinder about its centre and parallel to its length $= \frac{MR^2}{2}$

M.I. about its centre and perpendicular to its length $= M \left(\frac{L^2}{12} + \frac{R^2}{4} \right)$

According to problem, $\frac{ML^2}{12} + \frac{MR^2}{4} = \frac{MR^2}{2}$

By solving we get $L = \sqrt{3}R$

48 (d)



For circular disc 1

Mass = M , radius $R_1 = R$

Moment of inertia $I_1 = I_0$

For circular disc 2, of same thickness t , mass = M

But density = $\frac{1}{2} \times$ density of circular disc 1

Let radius = R_2

$$\text{Then } \pi R_2^2 t \times \frac{\rho}{2} = \pi R_1^2 t \times \rho = M \Rightarrow R_2^2 = 2R_1^2$$

$$R_2 = \sqrt{2}R_1 = \sqrt{2}R$$

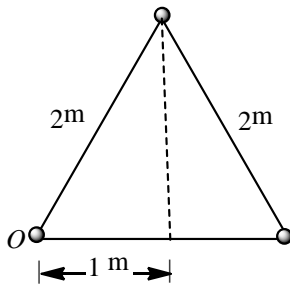
\therefore The given axis passes through the centre of mass

\therefore Moment of inertia $I \propto (\text{Radius})^2$

$$\Rightarrow \frac{I_1}{I_2} = \left(\frac{R_1}{R_2}\right)^2 \Rightarrow \frac{I_0}{I_2} = \left(\frac{R}{\sqrt{2}R}\right)^2 \Rightarrow I_2 = 2I_0$$

49 (b)

The x coordinate of centre of mass is



$$\begin{aligned} \bar{x} &= \frac{\sum m_i x_i}{\sum m_i} \\ &= \frac{m \times 0 + m \times 1 + m \times 2}{m + m + m} = 1 \end{aligned}$$

$$\begin{aligned} \bar{y} &= \frac{\sum m_i y_i}{\sum m_i} \\ &= \frac{m \times 0 + m(2 \sin 60^\circ) + m \times 0}{m + m + m} \end{aligned}$$

$$\bar{y} = \frac{\sqrt{3}m}{3m} = \frac{1}{\sqrt{3}}$$

Position vector of centre of mass is $\left(\hat{i} + \frac{\hat{j}}{\sqrt{3}}\right)$.

50 (c)

$$\alpha = \frac{2\pi(n_2 - n_1)}{t} = \frac{2\pi\left(0 - \frac{60}{60}\right)}{60} = \frac{-2\pi}{60} = \frac{-\pi}{30} \text{ rad/sec}^2$$

$$\therefore \tau = I\alpha = \frac{2 \times \pi}{30} = \frac{\pi}{15} \text{ N-m}$$

51 (b)

By doing so the distribution of mass can be made away from the axis of rotation



52 (d)

Moment of inertia of cylinder about an axis through the centre and perpendicular to its axis is

$$I_c = M \left(\frac{R^2}{4} + \frac{L^2}{12} \right)$$

Using theorem of parallel axes, moment of inertia of the cylinder about an axis through its edge would be

$$\begin{aligned} I &= I_c + M \left(\frac{L}{2} \right)^2 = M \left(\frac{R^2}{4} + \frac{L^2}{12} + \frac{L^2}{4} \right) \\ &= M \left(\frac{R^2}{4} + \frac{L^2}{3} \right) \end{aligned}$$

$$\text{When } L = 6R, I_h = \frac{49}{4} MR^2$$

53 (a)

Is perpendicular to the plane of the surface in which it moves

54 (a)

$$\frac{I_{\text{Sphere}}}{I_{\text{Cylinder}}} = \frac{\frac{2}{5} M_1 R^2}{\frac{1}{2} M_2 R^2} = \frac{\frac{2}{5} \left(\frac{4}{3} \pi R^3 \rho \right) R^2}{\frac{1}{2} (\pi R^2 L \rho) R^2} = \frac{16}{15}$$

$$\therefore I_{\text{Sphere}} > I_{\text{Cylinder}}$$

55 (d)

$$\mathbf{L} = \mathbf{r} \times \mathbf{P} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 2 & -1 \\ 3 & 4 & -2 \end{vmatrix}$$

$$\mathbf{L} = \hat{\mathbf{i}}(-4 + 4) - \hat{\mathbf{j}}(-2 + 3) + \hat{\mathbf{k}}(4 - 6) = -\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$$

\mathbf{L} has components along $-y$ axis and $-z$ axis.

The angular momentum is in $y - z$ plane *ie.*, perpendicular to x -axis.

56 (d)

$$I = MK^2 = 160 \Rightarrow K^2 = \frac{160}{M} = \frac{160}{10} = 16 \Rightarrow K = 4 \text{ metre}$$

57 (c)

$$\begin{aligned} \text{Loss of kinetic energy} &= \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (v_1 - v_2)^2 \\ &= \frac{1}{2} \frac{M \times M}{(M + M)} (v_1 - v_2)^2 \\ &= \frac{M \cdot M}{2(2M)} (v_1 - v_2)^2 \\ &= \frac{M}{4} (v_1 - v_2)^2 \end{aligned}$$

58 (c)

Let same mass and same outer radii of solid sphere and hollow sphere are M and R respectively. The moment of inertia of solid sphere A about its diameter

$$I_A = \frac{2}{5} MR^2 \quad \dots(i)$$

Similarly, the moment of inertia of hollow sphere (spherical shell) B about its diameter

$$I_B = \frac{2}{3} MR^2 \quad \dots(ii)$$

It is clear from Eqs. (i) and (ii), we get

$$I_A < I_B$$

59 (b)

$$I = MK^2 = \sum mR^2$$

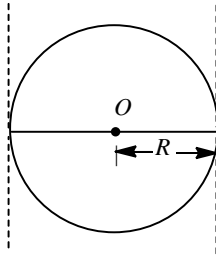
where M is the total mass of the body.

This means that

$$K = \sqrt{\left(\frac{I}{M}\right)}$$

According to thermo of parallel axis

$$I = I_{CG} + M(2R)^2$$



where, I_{CG} is moment of inertia about an axis through centre of gravity.

$$\therefore I = \frac{2}{5}MR^2 + 4MR^2 = \frac{22}{5}MR^2$$

$$\text{or } MK^2 = \frac{22}{5}MR^2$$

$$\therefore K = \sqrt{\frac{22}{5}}R$$

60 (c)

$$I = \frac{2}{5}MR^2 = \frac{2}{5}\left(\frac{4}{3}\pi R^3\rho\right)R^2 = \frac{8}{15}\times\frac{22}{7}R^5\rho$$

$$I = \frac{176}{105}R^5\rho$$

61 (a)

$$\frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right) = \frac{1}{2}(0.5)(0.2)^2\left(1 + \frac{2}{5}\right) = 0.014 J$$

62 (d)

$$\text{For disc, } I = \frac{1}{2}ma^2$$

$$\text{For ring, } I = ma^2$$

$$\text{For square of side } 2a = \frac{M}{12}[(2a)^2 + (2a)^2] = \frac{2}{3}Ma^2$$

For square of rod of length $2a$

$$I = 4\left[M\frac{(2a)^2}{12} + Ma^2\right] = \frac{16}{3}Ma^2$$

Hence, moment of inertia is maximum for square of four rods

63 (a)

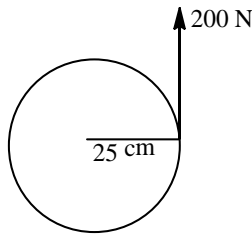
$$\tau = \frac{dL}{dt} = \frac{L_2 - L_1}{\Delta t} = \frac{5L - 2L}{3} = \frac{3L}{3} = L$$

64 (d)

$$L = \sqrt{2IE}. \text{ If } E \text{ are equal then } \frac{L_1}{L_2} = \sqrt{\frac{I_1}{I_2}} = \sqrt{\frac{I}{2I}} = \frac{1}{\sqrt{2}}$$

65 (a)

Clearly, the question refers to the torque about an axis through the centre of wheel. Then, since the radius to the point application of the force is the lever or momentum arm. we have



$$\tau = 0.25 \times 200 = 50 \text{ Nm}$$

66 (b)

$$\frac{I_{Ring}}{I_{Disc}} = \frac{MR^2}{1/2MR^2} = 2:1$$

67 (c)

Angular momentum

68 (a)

Initial acceleration of the system is zero. So it will always remain zero because there is no external force on the system

69 (b)

In the absence external force, position of centre of mass remain same therefore they will meet at their centre of mass

70 (b)

Time of descent will be less for solid sphere *i. e.* solid sphere will reach first at the bottom of inclined plane

71 (a)

$$\text{For solid sphere, } \frac{K^2}{R^2} = \frac{2}{5}$$

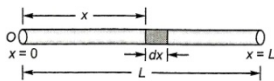
$$\text{For disc and solid cylinder, } \frac{K^2}{R^2} = \frac{1}{2}$$

As $\frac{K^2}{R^2}$ for solid sphere is smallest, it takes minimum time to reach the bottom of the incline

72 (b)

Let the mass of an element of length dx of rod located at a distance x away from left end is $\frac{M}{L} dx$. The x -coordinate of the centre of mass is given by

$$X_{CM} = \frac{1}{M} \int x dm$$



$$\begin{aligned} &= \frac{1}{M} \int_0^L x \left(\frac{M}{L} dx \right) \\ &= \frac{1}{L} \left[\frac{x^2}{2} \right]_0^L = \frac{L}{2} \end{aligned}$$

73 (c)

$$\begin{aligned} K &= \frac{L^2}{2I} = \frac{K_1}{K_2} = \frac{L_1^2}{L_2^2} \Rightarrow \frac{K_1}{K_2} = \left(\frac{100}{110} \right)^2 = \frac{100}{121} \\ \Rightarrow \frac{100}{K^2} &= \frac{100}{121} \Rightarrow K_2 = 121 = 100 + 21 \end{aligned}$$

Increase in kinetic energy = 21%

74 (c)

$$L = I\omega$$

75 (d)

Since, no external force is present on the system so, conservation principle of momentum is applicable

$$\therefore \vec{p}_i = \vec{p}_f = \vec{p}_1 + \vec{p}_2$$

$$\therefore \vec{p}_1 = -\vec{p}_2 \quad (\because \vec{p}_i = 0)$$

$$\therefore |\vec{p}_1| = |-\vec{p}_2|$$

$$\therefore \vec{p}_1 = \vec{p}_2$$

From this point of view, it is clear that momenta of both particles are equal in magnitude but opposite in direction

Also, friction is absent. So total mechanical energy of system remains conserved

76 (a)

In the absence of external torque angular momentum remains constant

77 (d)

$$v = \sqrt{\frac{2gh}{1 + \frac{I}{mr^2}}} = \sqrt{\frac{2 \times 10 \times 3}{1 + \frac{mr^2}{2 \times mr^2}}} = \sqrt{\frac{2 \times 10 \times 3}{\frac{3}{2}}} = \sqrt{40}$$

$$\Rightarrow v = r\omega$$

$$\Rightarrow r = \frac{v}{\omega} = \frac{\sqrt{40}}{2\sqrt{2}} = \sqrt{\frac{40}{8}} = \sqrt{5} \text{ m}$$

78 (b)

Moment of inertia of a circular ring about a diameter

$$I = \frac{1}{2} Mr^2$$

79 (d)

(a) Impulsive received by m

$$\vec{J} = m(\vec{v}_f - \vec{v}_i)$$

$$= m(-2\hat{i} + \hat{j} - 3\hat{i} - 2\hat{j})$$

$$= m(-5\hat{i} - \hat{j})$$

And impulse received by M

$$= -\vec{J} = m(5\hat{i} + \hat{j})$$

(b) $mv = m(5\hat{i} + \hat{j})$

Or $v = \frac{m}{M}(5\hat{i} + \hat{j}) = \frac{1}{13}(5\hat{i} + \hat{j})$

(c) $e = (\text{relative velocity of separation}/\text{relative velocity of approach})$ in the direction of $-\vec{J} = 11/17$

80 (b)

If rod is rotated about end A, then vertical component of velocity v_{\perp} of end A will be zero.

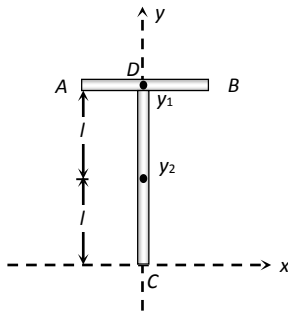
$$\begin{aligned} \therefore \omega &= \frac{v \cos 60^\circ}{l} = \frac{\sqrt{3}v}{2l} \\ &= \frac{\sqrt{3} \times 3}{2 \times 0.5} = 5.2 \text{ rads}^{-1} \end{aligned}$$

81 (a)

For translatory motion the force should be applied on the centre of the mass of the body. So we have to calculate the location of centre of mass of 'T' shaped object.

Let mass of rod AB is m so the mass of the rod CD will be $2m$

Let y_1 is the centre of mass of rod AB and y_2 is the centre of mass of rod CD. We can consider that whole mass of the rod is placed at their respective centre of mass *i. e.*, mass m is placed at y_1 and mass $2m$ is placed at y_2



Taking point 'C' at the origin position vector of point y_1 and y_2 can be written as $\vec{r}_1 = 2l\hat{j}$, $\vec{r}_2 = l\hat{j}$ and $m_1 = m$ and $m_2 = 2m$

Position vector of centre of mass of the system

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} = \frac{m2l\hat{j} + 2ml\hat{j}}{m + 2m} = \frac{4ml\hat{j}}{3m} = \frac{4}{3}l\hat{j}$$

Hence the distance of centre of mass from C = $\frac{4}{3}l$

82 (a)

When a body rolls down without slipping along an inclined plane of inclination θ , it rotates about a horizontal axis through its centre of mass and also its centre of mass moves. Therefore, rolling motion may be regarded as a rotational motion about an axis through its centre of mass plus a translational motion of the centre of mass. As it rolls down, it suffers loss in gravitational potential energy provided translational energy due to frictional force is converted into rotational energy.

83 (c)

This is an example of elastic oblique collision. When a moving body collides obliquely with another identical body in rest, then during elastic collision, the angle of divergence will be 90°

84 (c)

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} = \frac{200 \times 10\hat{i} + 500 \times (3\hat{i} + 5\hat{j})}{200 + 500}$$

$$\vec{v}_{cm} = 5\hat{i} + \frac{25}{7}\hat{j}$$

85 (c)

From $t_1 = 0$ to $t_2 = 2t_0$ the external force acting on the combined system is $m_1g + m_2g$

\therefore Total change in momentum of system

$$= Ft = (m_1 + m_2)g2t_0$$

86 (a)

$$L = \sqrt{2EI} = \sqrt{2 \times 10 \times 8 \times 10^{-7}} = 4 \times 10^{-3} \text{ kg m}^2/\text{s}$$

87 (c)

There is a point in the system, where if whole mass of the system is supposed to be concentrated, the nature of the motion executed by the system remains unaltered when the force acting on the system are applied directly at this point.

The position of centre of mass of system for n particles is expressed as

$$R_{CM} = \frac{\sum m_i r_i}{\sum m_i}$$

or $\sum m_i r_i = \text{constant}$

Hence, for a system having particles, we have

$$m_1 r_1 = m_2 r_2$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{m_2}{m_1}$$

ie, the centre of mass of a system of two particle divides the distance between them in inverse ratio of masses of particles.

88 (a)

The moment of inertia of ring = MR^2

The moment of inertia of removed sector = $\frac{1}{4}MR^2$

The moment of inertia of remaining part = $MR^2 - \frac{1}{4}MR^2$
 $= \frac{3}{4}MR^2$

According to question, the moment of inertia of the remaining part = kMR^2

then, $k = \frac{3}{4}$

89 (d)

Linear kinetic energy = $\frac{1}{2}mv^2$

Rotational kinetic energy = $\frac{1}{2}I\omega^2$
 $= \frac{1}{2}\left(\frac{2}{5}mr^2\right)\frac{v^2}{r^2}$
 $= \frac{1}{5}mv^2$

Total KE = $\frac{1}{2}mv^2 + \frac{1}{5}mv^2$
 $= \frac{7}{10}mv^2$

Required fraction = $\frac{\frac{1}{5}mv^2}{\frac{7}{10}mv^2}$
 $= \frac{1}{5} \times \frac{10}{7} = \frac{2}{7}$

90 (b)

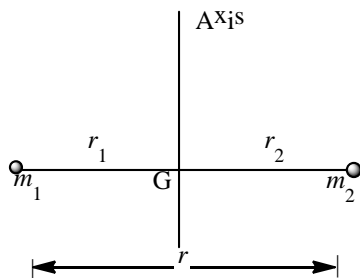
Moment of inertia of a rod about one end = $\frac{ML^2}{3}$

As, $I = I_1 + I_2 + I_3$

$\therefore I = 0 + \frac{ML^2}{3} + \frac{ML^2}{3} = \frac{2ML^2}{3}$

91 (d)

Total moment of inertia will be equal to the sum of moment of inertia due to individual masses.



$$I = \sum_{i=1}^n I_i$$

where, $I_1 = m_1 r_1^2, I_2 = m_2 r_2^2$.

Given, $r = r_1 + r_2$

$$m_1 r_1 = m_2 r_2$$

$$\therefore m_1 r_1 = m_2 (r - r_1)$$

$$\Rightarrow m_1 r_1 + m_2 r_1 = m_2 r$$

$$\Rightarrow r_1 (m_1 + m_2) = m_2 r$$

$$\Rightarrow r_1 = \frac{m_2 r}{(m_1 + m_2)}$$

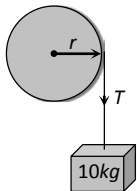
Also, $r_2 = r - r_1$
 $r_2 = r - \frac{m_2 r}{(m_1 + m_2)} = \frac{m_1 r}{m_1 + m_2}$
 $\therefore I = I_1 + I_2 = m_1 r_1^2 + m_2 r_2^2$
 $I = m_1 \frac{m_2^2 r^2}{(m_1 + m_2)^2} + m_2 \frac{m_1^2 r^2}{(m_1 + m_2)^2}$
 $I = \frac{m_1 m_2 r^2}{(m_1 + m_2)} = \frac{m_1 m_2}{(m_1 + m_2)} \cdot r^2$

92 (b)

$I = mR^2 = m \left(\frac{D^2}{4} \right) \Rightarrow I \propto mD^2$ or $m \propto \frac{I}{D^2}$
 $\therefore \frac{m_1}{m_2} = \frac{I_1}{I_2} \times \left(\frac{D_2}{D_1} \right)^2 = \frac{2}{1} \left(\frac{1}{2} \right)^2 = \frac{2}{4} = \frac{1}{2}$

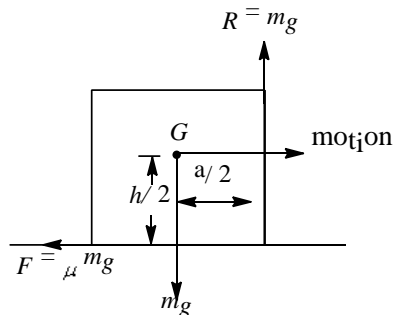
93 (d)

$\tau = r \times F$
 $= r \times T$
 $= r \times m \times g$
 $= 0.1 \times 10 \times 9.8$
 $= 9.8 \text{ N-m}$



94 (b)

As shown in figure normal reaction $R = mg$. Frictional force $F = \mu R = \mu mg$. To topple, clockwise moment must be more than the anticlockwise moment *ie*, $\mu mg \times \frac{h}{2} > mg \times \frac{a}{2}$ or $\mu > a/h$



95 (d)

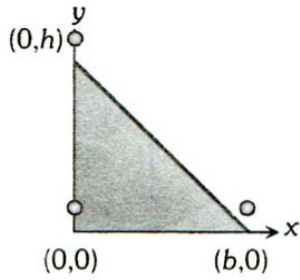
$\vec{v}_{cm} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2} = \frac{2 \times 3 + 3 \times 2}{2 + 3} = \frac{12}{5} = 2.4 \text{ m/s}$

96 (c)

We can assume that three particles of equal mass m are placed at the corners of triangle

$\vec{r}_1 = 0\hat{i} + 0\hat{j}, \vec{r}_2 = b\hat{i} + 0\hat{j}$
 and $\vec{r}_3 = 0\hat{i} + h\hat{j}$

$\therefore \vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3}{m_1 + m_2 + m_3}$



$$= \frac{b}{3} \hat{i} + \frac{h}{3} \hat{j}$$

i. e. coordinates of centre of mass is $(\frac{b}{3}, \frac{h}{3})$

97 (d)

Melting of ice produces water which will spread over larger distance away from the axis of rotation. This increases the moment of inertia so angular velocity decreases

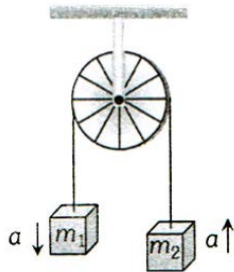
98 (a)

Acceleration of each mass = $a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$

Now acceleration of centre of mass of the system

$$A_{cm} = \frac{m_1 \vec{a}_1 + m_2 \vec{a}_2}{m_1 + m_2}$$

As both masses move with same acceleration but in opposite direction so $\vec{a}_1 = -\vec{a}_2 = a$ (let)



$$\therefore A_{cm} = \frac{m_1 a - m_2 a}{m_1 + m_2}$$

$$= \left(\frac{m_1 - m_2}{m_1 + m_2}\right) \times \left(\frac{m_1 - m_2}{m_1 + m_2}\right) \times g = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 \times g$$

99 (d)

Liner momentum = Moment of inertia \times angular velocity

100 (a)

$$P = \sqrt{p_x^2 + p_y^2}$$

$$= \sqrt{(2 \cos t)^2 + (2 \sin t)^2} = 2$$

If m be the mass of the body, then kinetic energy

$$= \frac{p^2}{2m} = \frac{(2)^2}{2m} = \frac{2}{m}$$

Since kinetic energy does not change with time, both work done and power are zero

Now Power = $Fv \cos \theta = 0$

As $F \neq 0, v \neq 0$

$\therefore \cos \theta = 0$

Or $\theta = 90^\circ$

As direction of \vec{p} is same that \vec{v} ($\vec{p} = m\vec{v}$) hence angle between \vec{F} and \vec{p} is equal to 90°

101 (b)

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2 = 2 \times 2 + \frac{1}{2} \times 3 \times (2)^2 = 10 \text{ rad}$$

102 (c)

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2 \Rightarrow 200 = \frac{1}{2} \alpha (5)^2 \Rightarrow \alpha = 16 \text{ rad/s}^2$$

103 (c)

Conservation of angular momentum

$$I_1 \omega_1 + I_2 \omega_2 = (I_1 + I_2) \omega$$

$$\text{Angular velocity of system } \omega = \frac{I_1 \omega_1 + I_2 \omega_2}{I_1 + I_2}$$

$$\therefore \text{Rotational kinetic energy} = \frac{1}{2} (I_1 + I_2) \omega^2$$

$$= \frac{1}{2} (I_1 + I_2) \left(\frac{I_1 \omega_1 + I_2 \omega_2}{I_1 + I_2} \right)^2 = \frac{(I_1 \omega_1 + I_2 \omega_2)^2}{2(I_1 + I_2)}$$

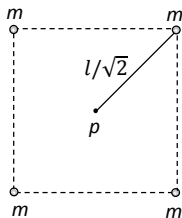
104 (a)

Moment of inertia of system about point P

$$= 4m \left(\frac{l}{\sqrt{2}} \right)^2 = 2ml^2$$

$$\text{and } 4mK^2 = 2ml^2$$

$$\therefore K = \frac{l}{\sqrt{2}}$$



105 (b)

$$E = \frac{L^2}{2I} \therefore E \propto L^2 \Rightarrow \frac{E_2}{E_1} = \left(\frac{L_2}{L_1} \right)^2$$

$$\frac{E_2}{E_1} = \left[\frac{L_1 + 200\% \text{ of } L_1}{L_1} \right]^2 = \left[\frac{L_1 + 2L_1}{L_1} \right]^2 = (3)^2 \Rightarrow E_2 = 9E_1$$

$$\text{Increment in kinetic energy } \Delta E = E_2 - E_1 = 9E_1 - E_1$$

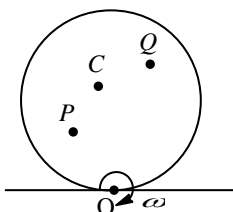
$$\Delta E = 8E_1 \therefore \frac{\Delta E}{E_1} = 8 \text{ or percentage increase} = 800\%$$

106 (b)

In free space neither acceleration due to gravity nor external torque act on the rotating solid space. Therefore, taking the same mass of sphere if radius is increased then moment of inertia, rotational kinetic energy and angular velocity will change but according to law of conservation of momentum, angular momentum will not change.

107 (a)

In case of pure rolling bottommost point is the instantaneous centre of zero velocity.



Velocity of any point on the disc, $v = r\omega$,
where r is distance of point from O .

$$r_Q > r_C > r_P$$

$$\therefore v_Q > v_C > v_P$$

108 (a)

Decreases his moment of inertia

109 (d)

Here, torque $\tau = 1.6 \times 1 = 1.6 \text{ Nm}$

So, when $d = 0.4 \text{ m}$,

$$F = \frac{\tau}{d} = \frac{1.6}{0.4} = 4 \text{ N}$$

110 (a)

$$\omega = \frac{v}{r} \Rightarrow \omega \propto \frac{1}{r}$$

111 (b)

If radius of earth decreases then its M.I. decreases

$$\text{As } L = I\omega \therefore \omega \propto \frac{1}{I} \text{ [} L = \text{constant]}$$

i. e. angular velocity of the earth will increase

112 (d)

$$\theta = \omega t = \frac{500 \times 2\pi}{60} = \frac{50\pi}{3} \text{ rad}$$

113 (d)

Collision of a bullet with a wooden block

114 (d)

Both the discs will reach at the same time

115 (c)

According to the principle of conservation of angular momentum, in the absence of external torque, the total angular momentum of the system is constant.

116 (a)

Initial angular momentum of ring, $L = I\omega = Mr^2\omega$

Final angular momentum of ring and four particles system

$$L = (Mr^2 + 4mr^2)\omega'$$

As there is no torque on the system therefore angular momentum remains constant

$$Mr^2\omega = (Mr^2 + 4mr^2)\omega' \Rightarrow \omega' = \frac{M\omega}{M + 4m}$$

117 (a)

$$F_1x + F_2x = F_3x$$

$$F_3 = F_1 + F_2$$

118 (b)

$$K_R = 50\%, K_T$$

$$\frac{1}{2}mv^2 \left(\frac{K^2}{R^2} \right) = \frac{50}{100} \times \frac{1}{2}mv^2 \Rightarrow \therefore \frac{K^2}{R^2} = \frac{1}{2}$$

i. e. body will be solid cylinder

119 (d)

Moment of inertia of a circular disc about an axis passing through centre of gravity and perpendicular to its plane

$$I = \frac{1}{2} MR^2 \quad \dots(i)$$

From Eq.(i)

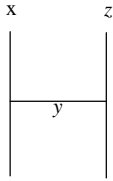
$$MR^2 = 2I$$

Then, moment of inertia of disc about tangent in a plane

$$= \frac{5}{4}MR^2 = \frac{5}{4}(2I) = \frac{5}{2}I$$

120 (d)

Moment of inertia of the system about rod x , figure



$$I = I_x + I_y + I_z = 0 + \left(\frac{Ml^2}{12} + \frac{Ml^2}{4} \right) + Ml^2$$

$$= \frac{4}{3}Ml^2$$

121 (b)

As net external force on the system is zero therefore position of their centre of mass remains unaffected i.e. they will hit each other at the point of centre of mass.

The centre of mass of the system lies nearer to A because $m_A > m_B$

122 (b)

Here, $r = 0.5$ m, $m = 2$ kg

$$\text{Rotational KE} = \frac{1}{2}I\omega^2 = \frac{1}{2} \times \left(\frac{1}{2}mr^2 \right) \omega^2$$

$$4 = \frac{1}{4}mv^2 = \frac{1}{4} \times 2v^2$$

$$v = \sqrt{8} = 2\sqrt{2} \text{ ms}^{-1}$$

123 (a)

Hollow cylinder will take more time to reach the bottom because it possess larger moment of inertia

124 (b)

Angular momentum = linear momentum \times Perpendicular distance of line of action of linear momentum from the axis of rotation = $mv \times l$

125 (c)

Momentum of 6 kg piece p_2 = momentum of 3 kg piece

$$p_1 = m_1v_1 = 3 \times 16 = 48 \text{ kg ms}^{-1}$$

$$\text{Kinetic energy of 6 kg piece } K_2 = \frac{p_2^2}{2m_2} = \frac{4 \times 48}{2 \times 8} = 192 \text{ J.}$$

126 (d)

$$Mv = \frac{M}{4}v_1 + \frac{3}{4}Mv_2$$

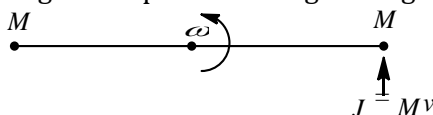
$$Mv = \frac{3}{4}Mv_2 \quad (\because v_1 = 0)$$

$$v_2 = \frac{4v}{3}$$

127 (a)

Let ω be the angular velocity of the rod. Applying,

Angular impulse = change in angular momentum about centre of mass of the system



$$J \cdot \frac{L}{2} = I_c \omega$$

$$\therefore (Mv) \left(\frac{L}{2}\right) = (2) \left(\frac{ML^2}{4}\right) \cdot \omega$$

$$\therefore \omega = \frac{v}{L}$$

128 (d)

Angle turned by the body

$$\theta = \theta_0 + \theta_1 t + \theta_2 t^2$$

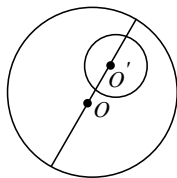
$$\begin{aligned} \text{Angular velocity } \omega &= \frac{d\theta}{dt} \\ &= \frac{d}{dt}(\theta_0 + \theta_1 t + \theta_2 t^2) \\ &= \theta_1 + 2\theta_2 t \end{aligned}$$

$$\begin{aligned} \text{Angular acceleration } \alpha &= \frac{d\omega}{dt} \\ &= \frac{d}{dt}(\theta_1 + 2\theta_2 t) \\ &= 2\theta_2 \end{aligned}$$

129 (a)

For the calculation of the position of centre of mass, cut off mass is taken as negative. The mass of disc is

$$m_1 = \pi r_1^2 \sigma$$



$$= \pi(6)^2 \sigma = 36\pi\sigma$$

Where σ is surface mass density

The mass of cutting portion is

$$m_2 = \pi(1)^2 \sigma = \pi\sigma$$

$$x_{\text{CM}} = \frac{m_1 x_1 - m_2 x_2}{m_1 - m_2}$$

Taking origin at the centre of disc,

$$x_1 = 0, x_2 = 3 \text{ cm}$$

$$x_{\text{CM}} = \frac{36\pi\sigma \times 0 - \pi\sigma \times 3}{36\pi\sigma - \pi\sigma} = \frac{-3\pi\sigma}{35\pi\sigma} = -\frac{3}{35} \text{ cm}$$

130 (c)

Distance between the centre of spheres = $12R$

$$\therefore \text{Distance between their surfaces} = 12R - (2R + R) = 9R$$

Since there is no external force, hence centre of mass must remain unchanged and hence

$$\Rightarrow m_1 r_1 = m_2 r_2 \Rightarrow Mx = 5M(9R - x) \Rightarrow x = 7.5R$$

131 (a)

$$15m + 10m = mv_1 + mv_2$$

$$25 = v_1 + v_2$$

$$\text{And } \frac{v_2 - v_1}{u_1 - u_2} = 1$$

$$\Rightarrow \frac{v_2 - v_1}{15 - 10} = 1$$

$$\Rightarrow v_2 - v_1 = 5$$

Solving Eqs. (i) and (ii), we have

$$v_2 = 15 \text{ ms}^{-1}, v_1 = 10 \text{ ms}^{-1}$$



132 (a)

Linear momentum p and kinetic energy K are interrelated as

$K = \frac{p^2}{2m}$ or $p = \sqrt{2mK}$, hence zero momentum implies zero kinetic energy and *vice versa*.

133 (a)

$$\omega_1 = \frac{900}{60} \times 2\pi = 30\pi \text{ rad/s}, \omega_f = 0, t = 60\text{s}$$

$$\omega_f = \omega_t + \alpha t \therefore \alpha = \frac{\omega_f - \omega_t}{t} = \frac{0 - 30\pi}{60} = -\frac{\pi}{2} \text{ rad/s}^2$$

134 (d)

Torque acting on a body in circular motion is zero.

135 (a)

Here, $m_1 = 1 \text{ kg}, \vec{v}_1 = 2\hat{i}$

$m_2 = 2 \text{ kg}, \vec{v}_2 = 2 \cos 30^\circ \hat{i} - 2 \sin 30^\circ \hat{j}$

$$\begin{aligned} \vec{v}_{\text{CM}} &= \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2} \\ &= \frac{1 \times 2\hat{i} + 2(2 \cos 30^\circ \hat{i} - 2 \sin 30^\circ \hat{j})}{1 + 2} \\ &= \frac{2\hat{i} + 2\sqrt{3}\hat{i} - 2\hat{j}}{3} = \left(\frac{2 + 2\sqrt{3}}{3}\right)\hat{i} - \frac{2}{3}\hat{j} \end{aligned}$$

136 (b)

According to theorem of parallel axes, moment of inertia of a rod about one of its ends

$$= \frac{Ml^2}{12} + \frac{Ml^2}{4} = \frac{Ml^2}{3}$$

\therefore Moment of inertia of two rods about Z-axis = Moment of inertia of 2 rods placed along X and Y-axis
 $= \frac{2Ml^2}{3}$

137 (c)

As per conservation law of momentum $0 = 4v + (A - 4)v_r$

$$\therefore \text{Recoil speed } v_r = \frac{4v}{(A-4)}$$

138 (b)

Mass distribution and axis of rotation

139 (a)

As we know that at the highest point, the shell has only the horizontal component of velocity which is $v \cos \theta$. If u be the velocity of second exploded piece, then applying conservation of linear momentum along x-axis

$$\therefore 2mv \cos \theta = -mv \cos \theta + mu$$

$$\text{Or } u = 3v \cos \theta$$

140 (d)

$$\text{M.I. of disc} = \frac{1}{2}mR^2 = \frac{1}{2}(\pi R^2 t)\rho R^2 = \frac{1}{2}\pi R^4 t\rho \quad [\rho = \text{density}, t = \text{thickness}]$$

If discs are made of same material and same thickness then $I \propto R^4 \propto (\text{Diameter})^4$

$$\therefore \frac{I_A}{I_B} = \left(\frac{D_A}{D_B}\right)^4 = \left(\frac{2}{1}\right)^4 = \frac{16}{1}$$

141 (c)

The kinetic energy of the solid sphere,

$$K = \frac{1}{2}Mv^2 \quad \dots(i)$$

The rotational kinetic energy,

$$K_r = \frac{1}{2} I \omega^2$$

But $I = \frac{2}{5} MR^2$ for solid sphere and $\omega = \frac{v}{R}$

then,
$$K_r = \frac{1}{2} \times \frac{2}{5} MR^2 \times \frac{v^2}{R^2}$$

$$K_r = \frac{1}{5} Mv^2 \quad \dots(ii)$$

On dividing Eq. (ii) by Eq. (i), we have

$$\frac{K_r}{K} = \frac{1/5 Mv^2}{1/2 Mv^2}$$

or
$$\frac{K_r}{K} = \frac{2}{5}$$

142 (c)

In rotational motion of a rigid body, the centre of mass of the body moves uniformly in a circular path.

143 (d)

No of revolution = Area of trapezium

$$= \frac{1}{2} \times (2.5 + 5) \times 3000 = 11250 \text{ rev}$$

144 (c)

$$\frac{3}{2} MR^2$$

145 (d)

Angular momentum

146 (b)

The centre of mass of the body remains unchanged

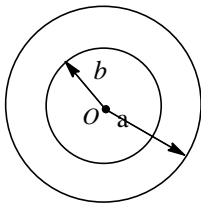
147 (a)

From conservation of momentum

$$Mv = m \times 0 + (M - m)v' \Rightarrow v' = \frac{Mv}{(M - m)}$$

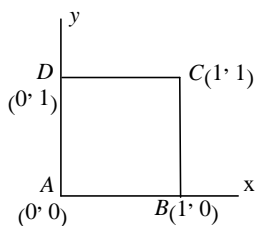
148 (c)

There is no shift in position of centre of mass, as clear from the figure



149 (c)

$$x_{CM} = \frac{m_A x_A + m_B x_B + m_C x_C + m_D x_D}{m_A + m_B + m_C + m_D}$$



$$= \frac{1 \times 0 + 2 \times 1 + 3 \times 1 + 4 \times 0}{1 + 2 + 3 + 4}$$

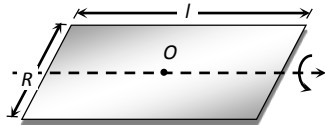
$$= \frac{2+3}{10} = \frac{1}{2} = 0.5 \text{ m}$$

Similarly,
$$y_{CM} = \frac{m_A y_A + m_B y_B + m_C y_C + m_D y_D}{m_A + m_B + m_C + m_D}$$

$$= \frac{1 \times 0 + 2 \times 0 + 3 \times 1 + 4 \times 1}{1 + 2 + 3 + 4}$$

$$= \frac{7}{10} = 0.7 \text{ m}$$

150 (d)



M.I. of plane about O and parallel to length = $\frac{Mb^2}{12}$

151 (c)

The impact force $F = \frac{\Delta p}{\Delta t} = v \frac{\Delta m}{\Delta t}$ where $\frac{\Delta m}{\Delta t}$ = rate of flow of water in the nozzle and v the velocity of water jet.

Since the ball is in equilibrium $F = mg$ where m = mass of ping pong ball.

$$\Rightarrow v \frac{\Delta m}{\Delta t} = mg \text{ or rate of flow of water } \frac{\Delta m}{\Delta t} = \frac{mg}{v}$$

152 (d)

$$L = I\omega$$

153 (d)

As the inclined plane is frictionless therefore all the bodies will slide down along the inclined plane with same acceleration $g \sin \theta$

154 (c)

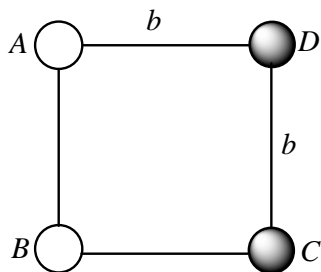
$$F = v \frac{\Delta m}{\Delta t}$$

$$\text{Here, } \frac{\Delta m}{\Delta t} = \frac{nm}{t} = \frac{120 \times 10 \times 10^{-3}}{60} = 20 \times 10^{-3} \text{ kg s}^{-1}$$

$$\therefore F = 800 \times 20 \times 10^{-3} \text{ N} = 16 \text{ N}$$

155 (c)

Moment of inertia of a hollow sphere of radius R about the diameter passing through D is



$$I_A = \frac{2}{5}MR^2 \quad \dots(i)$$

Moment of inertia of solid sphere about diameter

$$I_B = \frac{2}{5}MR^2 \quad \dots(ii)$$

\therefore Moment of inertia of whole system about side

$$AD = I_A + I_D + I_B + I_C$$

$$= \frac{2}{3}MR^2 + \frac{2}{5}MR^2 + \left(Mb^2 + \frac{2}{3}MR^2\right) + \left(Mb^2 + \frac{2}{5}MR^2\right)$$

$$= \frac{32}{15}MR^2 + 2Mb^2$$

156 (d)

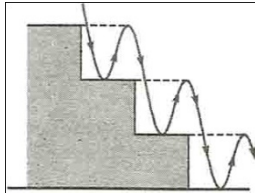
$$\tau = \mathbf{r} \times \mathbf{F}$$

τ is perpendicular to both \mathbf{r} and \mathbf{F} , so $\mathbf{r} \cdot \tau$ as well as $\mathbf{F} \cdot \tau$ has to be zero.

157 (c)

As shown in adjoining figure ball is falling from height $2h$ and rebounding to a height h only. It means that velocity of ball just before collision

$$u = \sqrt{\frac{2(2h)}{g}} = \sqrt{\frac{4h}{g}} \text{ and velocity just after collision}$$



$$v = -\sqrt{\frac{2h}{g}}$$

$$\therefore e = \frac{-v \sqrt{\frac{2h}{g}}}{u \sqrt{\frac{4h}{g}}} = \frac{1}{\sqrt{2}}$$

158 (b)

Centre of Mass of a solid body is given by

$$x_{CM} = \frac{\sum_{i=1}^n \Delta m_i x_i}{\sum_{j=1}^n \Delta M_j}$$

$$y_{CM} = \frac{\sum_{i=1}^n \Delta m_i y_i}{\sum_{j=1}^n \Delta M_j}$$

$$z_{CM} = \frac{\sum_{i=1}^n \Delta m_i z_i}{\sum_{j=1}^n \Delta M_j}$$

$$1 \times x_1 + 2 \times x_2 + 3 \times x_3 = (1 + 2 + 3)3 \quad \dots(i)$$

and $x_1 = x_2 = x_3 = 3$

$$x_{CM} = y_{CM} = z_{CM} = 1 \quad (\text{given})$$

$$1(1 + 2 + 3 + 4) = 1x_1 + 2x_2 + 3x_3 + 4x_4 \quad \dots(ii)$$

Solving Eqs. (i) and (ii), we get

$$4x_4 = 10 - 18$$

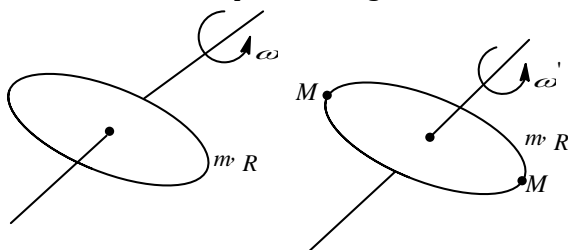
$$x_4 = -2$$

Similarly, $y_4 = -2, z_4 = -2$

The fourth particle must be placed at the point $(-2, -2, -2)$.

159 (d)

As no external torque is acting about the axis, angular momentum of system remains conserved.

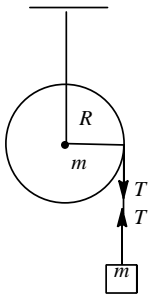


$$\therefore I_1 \omega = I_2 \omega'$$

$$\Rightarrow mR^2 \omega = (mR^2 + 2MR^2) \omega'$$

$$\Rightarrow \omega' = \left(\frac{m}{m+2M} \right) \omega$$

160 (b)



For the motion of the block

$$mg - T = ma \quad \dots(i)$$

For the rotation of the pulley

$$\tau = TR = I\alpha$$

$$\Rightarrow T = \frac{1}{2}mR\alpha \quad \dots(ii)$$

As string does not slip on the pulley

$$a = R\alpha \quad \dots(iii)$$

On solving Eqs. (i), (ii) and (iii)

$$a = \frac{2g}{3}$$

161 (b)

We know $m_1r_1 = m_2r_2 \Rightarrow m \times r = \text{constant} \therefore r \propto \frac{1}{m}$

Assertion - Reasoning Type

This section contain(s) 0 questions numbered 1 to 0. Each question contains STATEMENT 1(Assertion) and STATEMENT 2(Reason). Each question has the 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

- a) Statement 1 is True, Statement 2 is True; Statement 2 is correct explanation for Statement 1
b) Statement 1 is True, Statement 2 is True; Statement 2 is **not** correct explanation for Statement 1
c) Statement 1 is True, Statement 2 is False
d) Statement 1 is False, Statement 2 is True
- 1 **Statement 1:** A ladder is more apt to slip, when you are high up on it than when you just begin to climb
Statement 2: At the high up on a ladder, the torque is large and on climbing up the torque is small
- 2 **Statement 1:** When ice on polar caps of earth melts, duration of the day increases
Statement 2: $L = L\omega = I \cdot \frac{2\pi}{T} = \text{constant}$
- 3 **Statement 1:** Radius of gyration of body is a constant quantity
Statement 2: The radius of gyration of a body about an axis of rotation may be defined as the root mean square distance of the particle from the axis of rotation
- 4 **Statement 1:** The position of centre of mass of a body does not depend upon shape and size of the body
Statement 2: Centre of mass of a body lies always at the centre of the body
- 5 **Statement 1:** The velocity of a body at the bottom of an inclined plane of given height is more when it slides down the plane compared to when it rolls down the same plane
Statement 2: In rolling down, a body acquires both, KE of translation and KE of rotation
- 6 **Statement 1:** In rolling, all points of a rigid body have the same linear speed
Statement 2: The rotational motion does not affect the linear velocity of rigid body
- 7 **Statement 1:** The speed of whirlwind in a tornado is alarmingly high
Statement 2: If no external torque acts on a body, its angular velocity remains conserved
- 8 **Statement 1:** The centre of mass of a body will change with the change in shape and size of the body.
Statement 2: $\vec{r} = \frac{\sum_{i=1}^{i=n} m_i \vec{i}_i}{\sum_{i=1}^{i=n} m_i}$
- 9 **Statement 1:** A particle is moving on a straight line with a uniform velocity, its angular momentum is always zero
Statement 2: The momentum is zero when particle moves with a uniform velocity
- 10 **Statement 1:** At the centre of earth, a body has centre of mass, but no centre of gravity
Statement 2: Acceleration due to gravity is zero at the centre of earth
- 11 **Statement 1:** Inertia and moment of inertia are same quantities
Statement 2: Inertia represents the capacity of a body to oppose its state of motion or rest



- 12 **Statement 1:** The centre of mass of an electron and proton, when released moves faster towards proton
Statement 2: Proton is heavier than electron
- 13 **Statement 1:** The centre of mass of a proton and an electron, released from their respective positions remains at rest
Statement 2: The centre of mass remain at rest, if no external force is applied
- 14 **Statement 1:** The centre of mass of body may lie there is no mass.
Statement 2: Centre of mass of a body is a point, where the whole mass of the body is supposed to be con centrated
- 15 **Statement 1:** The centre of mass of an electron and proton, when released moves faster towards proton
Statement 2: This is because proton is heavier
- 16 **Statement 1:** There are two propellers in a helicopter
Statement 2: Angular momentum is conserved
- 17 **Statement 1:** The centre of mass of a body may lie where there is no mass
Statement 2: The centre of mass has nothing to do with the mass
- 18 **Statement 1:** The centre of mass of a body may lie where there is no mass
Statement 2: Centre of mass of a body is a point, where the whole mass of the body is supposed to be concentrated



: ANSWER KEY :

1)	a	2)	a	3)	d	4)	d	5)	b	6)	d	7)	c	8)	a
9)	d	10)	a	11)	d	12)	d	13)	a	14)	a	15)	d	16)	b
17)	b	18)	a												



: HINTS AND SOLUTIONS :

- 1 (a)
When a person is high up on the ladder, than a large torque is produced due to his weight about the point of contact between the ladder and the floor. Whereas when he starts climbing up. The torque is small. Due to this reason, the ladder is more apt to slip, when one is high up on it
- 2 (a)
Both, the assertion and reason are true and latter is a correct explanation of the former. Infact, as ice on polar caps of earth melts, mass near the polar axis spreads out, I increases. Therefore, T increases ie, duration of day increases
- 3 (d)
Radius of gyration of body is not a constant quantity. Its value changes with the change in location of the axis of rotation. Radius of gyration of a body about a given axis is given as
- $$K = \sqrt{\frac{r_1^2 + r_2^2 + \dots + r_n^2}{n}}$$
- 4 (d)
The position of centre of mass of a body depends on shape, size and distribution of mass of the body. The centre of mass does not lie necessarily at the centre of the body
- 5 (b)
In sliding down, the entire PE is converted only into linear KE. In rolling down, a part of same PE is converted into KE of rotation. Therefore, velocity acquired is less. Both the statements are true, but statement-2 is not a correct explanation of statement
- 6 (d)
In rolling all points of rigid body have the same angular speed but different linear speed
- 7 (c)
In a whirlwind in a tornado, the air from nearby regions gets concentrated in a small space thereby decreasing the value of its moment of inertia considerably. Since, $I\omega = \text{constant}$, so due to decrease in moment of inertia of the air, its angular speed increases to a high value
If no external torque acts, then
 $\tau = 0 \Rightarrow \frac{dL}{dt} = 0$ or $L = \text{constant} \Rightarrow I\omega = \text{constant}$
As in the rotational motion, the moment of inertia of the body can change due to the change in position of the axis of rotation, the angular speed may not remain conserved
- 8 (a)
Position vector of centre of mass depends on masses of particles and their location. Therefore, change in shape/size of body do change the centre of mass
- 9 (d)
When particle moves with constant velocity \vec{v} then its linear momentum has some finite value ($\vec{P} = m\vec{v}$)
Angular momentum (L) = Linear momentum (P) \times Perpendicular distance of line of action of linear momentum from the point of rotation (d)
So if $d \neq 0$ then $L \neq 0$, but if $d = 0$ then L may be zero. So we can conclude that angular momentum of a particle moving with constant velocity is not always zero

- 10 (a)
At the centre of earth, $g = 0$. Therefore a body has no weight at the centre of earth and have no centre of gravity (centre of gravity of a body is the point where the resultant force of attraction or the weight of the body acts). But centre of mass of a body depends on mass and position of particles and is independent of weight
- 11 (d)
There is a difference between inertia and moment of inertia of a body. The inertia of a body depends only upon the mass of the body but the moment of inertia of a body about an axis not only depends upon the mass of the body but also upon the distribution of mass about the axis of rotation
- 12 (d)
The position of centre of mass of electron and proton remains at rest. As their motion is due to internal force of electrostatic attraction, which is conservative force. No external force is acting on the two particles, therefore centre of mass remain at rest
- 13 (a)
Initially the electron and proton were at rest so their centre of mass will be at rest. When they move towards each other under mutual attraction then velocity of centre of mass remains unaffected because external force on the system is zero
- 14 (a)
As the concept of centre of mass is only theoretical, therefore in practice no mass may lie at the centre of mass. For example, centre of mass of a uniform circular ring is at the centre of the ring where there is no mass.
- 15 (d)
The position of centre of mass of electron and proton remains at rest, at their motion is due to (internal) forces of electrostatic attraction, which are conservative. No external force, what so ever is acting on the two particles
- 16 (b)
Both, assertion and reason are true but the reason is not a correct explanation of the statement-1. Infact, if helicopter has only one propeller, the helicopter itself would turn in opposite direction to conserve the angular momentum
- 17 (b)
The assertion and reason, both are true. But the reason is not a correct explanation of the assertion. Infact, the centre of mass is related to the distribution of mass of the body
- 18 (a)
As the concept of centre of mass is only theoretical, therefore in practice no mass may lie at the centre of mass. For example, centre of mass of a uniform circular ring is at the centre of the ring where there is no mass

Matrix Match Type

This section contain(s) 0 question(s). Each question contains Statements given in 2 columns which have to be matched. Statements (A, B, C, D) in **columns I** have to be matched with Statements (p, q, r, s) in **columns II**.

1. Each of the body in column I show the moment of inertia about its diameter in column II. Select the correct answer (matching list I with List II) as per code given below the lists.

Column-I		Column- II	
(A)	Ring	(1)	$\frac{M}{4}(R_1^2 + R_2^2)$
(B)	Disc	(2)	$\frac{1}{4}MR^2$
(C)	Annular Disc	(3)	$\frac{1}{2}MR^2$

CODES :

	A	B	C
a)	2	3	1
b)	3	2	1
c)	3	1	2
d)	1	2	3



: ANSWER KEY :

1)	b														
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: HINTS AND SOLUTIONS :

1 (b)

The moment of inertia of a ring about its diameter $= \frac{1}{2}MR^2$

The moment of inertia of a disc about its diameter $= \frac{1}{4}Ma^2$

The moment of inertia of an annular disc about its diameter $= \frac{1}{4}M(R_1^2 + R_2^2)$

