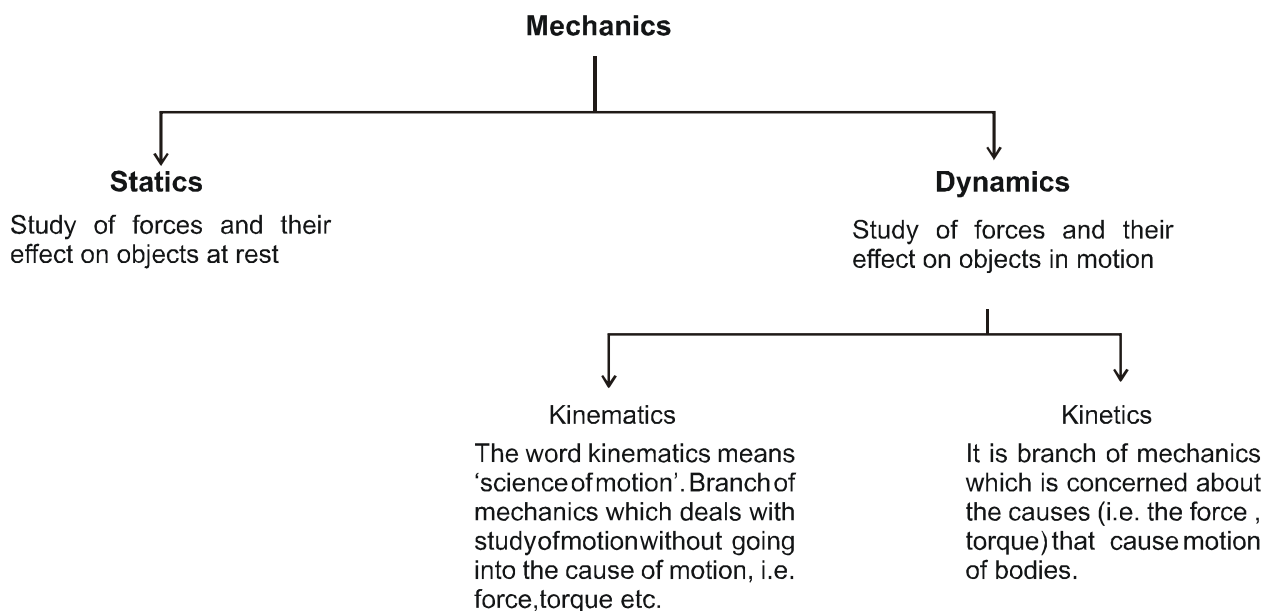


MOTION IN A STRAIGHT LINE

MECHANICS

Mechanics is the branch of physics which deals with the cause and effects of motion of a particle, rigid objects and deformable bodies etc. Mechanics is classified under two streams namely Statics and Dynamics. Dynamics is further divided into Kinematics and Kinetics.



1. MOTION AND REST

Motion is a combined property of the object and the observer. There is no meaning of rest or motion without the observer. Nothing is in absolute rest or in absolute motion.

An object is said to be in motion with respect to an observer, if its position changes with respect to that observer. It may happen by both ways either observer moves or object moves.

2. RECTILINEAR MOTION

Rectilinear motion is motion, along a straight line or in one dimension. It deals with the kinematics of a particle in one dimension.

For example the motion of an ant on a wire is a rectilinear motion.

(i) Position :

The position of a particle refers to its location in the space at a certain moment of time. It is concerned with the question – “*where is the particle at a particular moment of time?*”

(ii) Displacement :

The shortest distance from the initial position to the final position of the particle is called displacement. The displacement of a particle is measured as the change in the position of the particle in a particular direction over a given time interval. It depends only on final & initial positions.

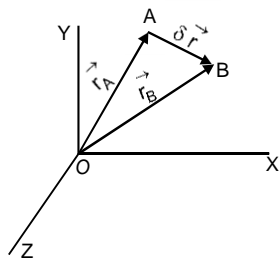
Displacement of a particle is a position vector of its final position w.r.t. initial position.

Position vector of A w.r.t. O = \overline{OA}

$$\Rightarrow \vec{r}_A = x_1 \hat{i} + y_1 \hat{j} + z_1 \hat{k}$$

Position vector of B w.r.t. O = \overline{OB}

$$\Rightarrow \vec{r}_B = x_2 \hat{i} + y_2 \hat{j} + z_2 \hat{k}$$



$$\text{Displacement} = \overline{AB} = (x_2 - x_1) \hat{i} + (y_2 - y_1) \hat{j} + (z_2 - z_1) \hat{k}$$

Characteristics of Displacement :

- (i) It is a vector quantity.
- (ii) The displacement of a particle between any two points is equal to the shortest distance between them.
- (iii) The displacement of an object in a given time interval can be positive, negative or zero.
- (iv) Dimension : $[M^0L^1T^0]$
- (v) Unit : In C.G.S. centimeter (cm), In S.I. system meter (m).

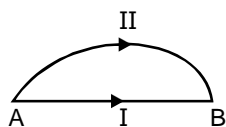
(iii) Distance :

The length of the actual path between initial & final positions of a particle in a given interval of time is called distance covered by the particle. Distance is the actual length of the path. It is the characteristic property of any path i.e., path is always associated when we consider distance between two positions.

Distance between A & B while moving through path (1) may or may not be equal to the distance between A & B while moving through path (2)

Characteristics of Distance :

- (i) It is a scalar quantity.
- (ii) It depends on the path.



- (iii) It never reduces with time.
- (iv) Distance covered by a particle is always positive & can never be negative or zero.
- (v) Dimension : $[M^0 L^1 T^0]$
- (vi) Unit : In C.G.S.system centimeter (cm), In S.I. system metre (m).

COMPARATIVE STUDY OF DISPLACEMENT AND DISTANCE

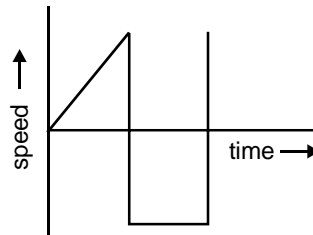
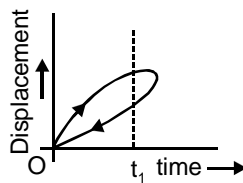
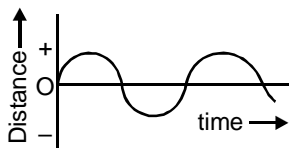
S.No.	Displacement	Distance
1.	It has single value between two points	It may have more than one value between two points
2.	$0 \leq \text{Displacement} \leq 0$	Distance > 0
3.	Displacement can decrease with time	It can never decrease with time.
4.	It is a vector quantity	It is a scalar quantity

Special Note

The actual distance travelled by a particle in the given interval of time is always equal to or greater than the magnitude of the displacement and in no case, it is less than the magnitude of the displacement, i.e., $\text{Distance} \geq |\text{Displacement}|$

- Displacement may be + ve, – ve or zero.
- Distance, speed and time can never be negative.
- At the same time particle cannot have two positions.

SOME IMPOSSIBLE GRAPHS :

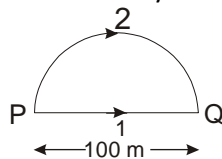


Solved Examples

Solved Examples

Example 1. Ram takes path 1 (straight line) to go from P to Q and Shyam takes path 2 (semicircle).

- Find the distance travelled by Ram and Shyam?



- Find the displacement of Ram and Shyam?

Solution

Distance travelled by Ram = 100 m

Distance travelled by Shyam = $\pi(50 \text{ m}) = 50\pi \text{ m}$

Displacement of Ram = 100 m

Displacement of Shyam = 100 m

Example 2. An old person moves on a semi circular track of radius 40 m during a morning walk. If he starts at one end of the track and reaches at the other end. Find the displacement of the person.

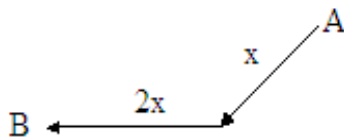
Solution Displacement = $2R = 2 \times 40 = 80$ meter.

Example 3. An athlete is running on a circular track of radius 50 meter. Calculate the displacement of the athlete after completing 5 rounds of the track.

Solution Since final and initial positions are same .

Hence displacement of athlete will be $\Delta r = r - r = 0$

Example 4. If a particle moves from point A to B then distance covered by particle will be.



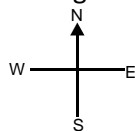
Solution $D = x + 2x = 3x$

Example 5. A monkey is moving on circular path of radius 80 m. Calculate the distance covered by the monkey.

Solution Distance = Circumference of the circle

$$D = 2\pi R \Rightarrow D = 2\pi \times 80 = 160 \times 3.14 = 502.40\text{m}$$

Example 6. A man has to go 50 m due north, 40 m due east and 20 m due south to reach a field.

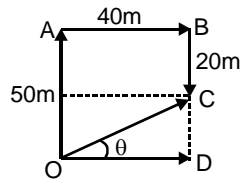


(a) What distance he has to walk to reach the field?

(b) What is his displacement from his house to the field?

Solution Let origin be O then

$$\begin{aligned} \text{(a) Distance covered} &= OA + AB + BC \\ &= 50 + 40 + 20 \\ &= 110 \text{ meter} \end{aligned}$$

(b) **First method :**

$$\begin{aligned} \text{Displacement } OC &= \sqrt{OD^2 + CO^2} \\ &= \sqrt{40^2 + 30^2} \\ &= 10\sqrt{25} = 50 \text{ meter} \end{aligned}$$

Second method :

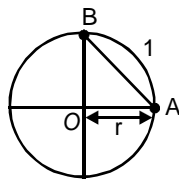
$$\begin{aligned} \text{Displacement } \vec{d} &= 50\hat{j} + 40\hat{i} - 20\hat{j} \\ &= 30\hat{j} + 40\hat{i} \\ |\vec{d}| &= \sqrt{40^2 + 30^2} = 50 \text{ meter} \end{aligned}$$

Example 7. A body covers $\frac{1}{4}$ th part of a circular path. Calculate the ratio of distance and displacement.

Solution

Distance = AB from path (1)

Displacement = AB



$$\begin{aligned} &= \frac{2\pi r}{4} = \frac{\pi r}{2} & &= \sqrt{OA^2 + OB^2} \\ & & &= \sqrt{r^2 + r^2} = r\sqrt{2} \end{aligned}$$

$$\therefore \frac{\text{Distance}}{\text{Displacement}} = \frac{\pi r / 2}{r\sqrt{2}} = \frac{\pi}{2\sqrt{2}}$$

**(iv) Speed :**

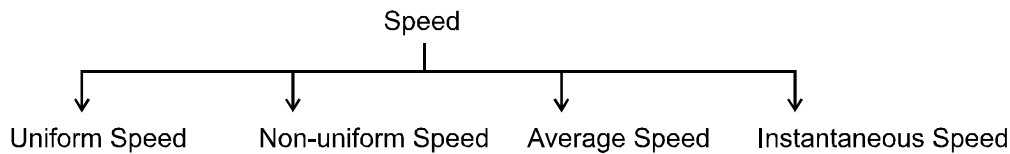
Speed of an object is defined as the time rate of change of position of the object in any direction. It is measured by the distance travelled by the object in unit time in any direction. i.e.,

$$\text{speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

- (i) It is a scalar quantity.
- (ii) It gives no idea about the direction of motion of the object
- (iii) It can be zero or positive but never negative.
- (iv) Unit: C.G.S. cm/sec, S.I. m/sec,

$$1 \text{ km/h} = \frac{1000}{60 \times 60} = \frac{5}{18} \text{ m/s} \quad 1 \text{ km/h} = \frac{5}{18} \text{ m/s}$$

- (v) Dimension : $M^0 L^1 T^{-1}$

Types of speed :**i. Uniform speed :**

An object is said to be moving with a uniform speed, if it covers equal distances in equal intervals of time, howsoever small these intervals may be. The uniform speed is shown by straight line in distance-time graph.

For example: suppose a train travels 1000 metre in 60 second. The train is said to be moving with uniform speed, if it travels 500 metre in 30 second, 250 metre in 15 second, 125 metre in 7.5 second and so on.

ii. Non Uniform Speed :

An object is said to be moving with a variable speed if it covers equal distance in unequal intervals of time or unequal distances in equal intervals of time, howsoever small these intervals may be.

For example : suppose a train travels first 1000 metre in 60 second, next 1000 metre in 120 second and next 1000 metre in 50 second, then the train is moving with variable speed.

iii. Average Speed :

When an object is moving with a variable speed, then the average speed of the object is that constant speed with which the object covers the same distance in a given time as it does while moving with variable speed during the given time. Average speed for the given motion is defined as the ratio of the total distance travelled by the object to the total time taken i.e.,

$$\text{Average speed } \bar{V} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

Note : If any car covers distance x_1, x_2, \dots in the time intervals t_1, t_2, \dots then.

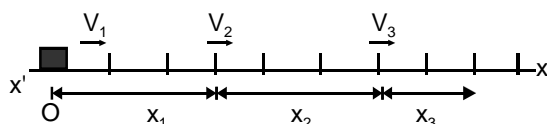
$$\bar{V} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{t_1 + t_2 + \dots + t_n}$$

SOME IMPORTANT CASES RELATED TO AVERAGE SPEED :**Case : 1**

If body covers distances x_1 , x_2 , and x_3 with speeds v_1 , v_2 , and v_3 respectively in same direction then average speed of body.

$$\Rightarrow \bar{V} = \frac{x_1 + x_2 + x_3}{t_1 + t_2 + t_3} \quad \text{here, } t_1 = \frac{x_1}{v_1}, t_2 = \frac{x_2}{v_2}, t_3 = \frac{x_3}{v_3}$$

$$\bar{V} = \frac{x_1 + x_2 + x_3}{\frac{x_1}{v_1} + \frac{x_2}{v_2} + \frac{x_3}{v_3}}$$



If body covers equal distances with different speeds then, $x_1 = x_2 = x_3 = x$

$$\bar{V} = \frac{3x}{\frac{x}{v_1} + \frac{x}{v_2} + \frac{x}{v_3}} = \frac{3}{\frac{1}{v_1} + \frac{1}{v_2} + \frac{1}{v_3}} = \frac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_3v_1}$$

Case : 2

If any body travels with speeds v_1, v_2, v_3 during time intervals t_1, t_2, t_3 respectively then the average speed of the body will be

$$\text{Average speed } \bar{V} = \frac{x_1 + x_2 + x_3}{t_1 + t_2 + t_3} = \frac{v_1t_1 + v_2t_2 + v_3t_3}{t_1 + t_2 + t_3} \quad \text{If } t_1 = t_2 = t_3 = t$$

$$= \frac{(v_1 + v_2 + v_3) \times t}{3 \times t} = \frac{(v_1 + v_2 + v_3)}{3}$$

(d) Instantaneous speed :

The speed of the body at any instant of time or at a particular position is called instantaneous speed.

Let a body travel a distance Δx in the time interval Δt , then its average speed = $\frac{\Delta x}{\Delta t}$.

When $\Delta t \rightarrow 0$, then average speed of the body becomes the instantaneous speed.

(v) Velocity

It is defined as rate of change of displacement.

Characteristics of Velocity :

- i. It is a vector quantity.
- ii. Its direction is same as that of displacement.
- iii. Unit and dimension : Same as that of speed.

Types of Velocity :

- a) Instantaneous Velocity
- b) Average Velocity
- c) Uniform Velocity
- d) Non-uniform Velocity

(a) Instantaneous Velocity : It is defined as the velocity at some particular instant.

$$\text{Instantaneous velocity} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

(b) Average Velocity : Average Velocity = $\frac{\text{Total Displacement}}{\text{Total time}}$

(c) Uniform Velocity : A particle is said to have uniform velocity, if magnitudes as well as direction of its velocity remains same and this is possible only when the particles moves in same straight line reserving its direction.

(d) Non-uniform Velocity : A particle is said to have non-uniform velocity, if either of magnitude or direction of velocity changes (or both changes).

COMPARATIVE STUDY OF SPEED AND VELOCITY

S.No.	Speed	Velocity
1.	It is the time rate of change of distance of a body. particle	It is the time rate of change of displacement of a particle
2.	It tells nothing about the direction of motion of the particle	It tells the direction of motion of the particle
3.	It can be positive or zero	It can be positive or negative or zero
4.	It is a scalar quantity	It is a vector quantity

Note :

The magnitude of instantaneous velocity and instantaneous speed are equal.

The determination of instantaneous velocity by using the definition usually involves calculation of derivative. We can find $v = \frac{dx}{dt}$ by using the standard results from differential calculus.

It is always tangential to the path.

Average speed is always positive in contrast to average velocity which being a vector, can be positive or negative.

If the motion of a particle is along a straight line and in same direction then, average velocity = average speed.

Average speed is, in general, greater than the magnitude of average velocity.

Solved Examples

Solved Examples

Example 1. In the example 1, if Ram takes 4 seconds and Shyam takes 5 seconds to go from P to Q, find

- (a) Average speed of Ram and Shyam?
 (b) Average velocity of Ram and Shyam?

Solution :

(a) Average speed of Ram = $\frac{100}{4}$ m/s = 25 m/s

Average speed of Shyam = $\frac{50\pi}{5}$ m/s = 10π m/s

(b) Average velocity of Ram = $\frac{100}{4}$ m/s = 25 m/s (From P to Q)

Average velocity of Shyam = $\frac{100}{5}$ m/s = 20 m/s (From P to Q)

Example 2. A particle travels half of total distance with speed v_1 and next half with speed v_2 along a straight line. Find out the average speed of the particle?

Solution : Let total distance travelled by the particle be $2s$.

Time taken to travel first half = $\frac{s}{v_1}$

Time taken to travel next half = $\frac{s}{v_2}$

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}} = \frac{2s}{\frac{s}{v_1} + \frac{s}{v_2}} = \frac{2v_1v_2}{v_1 + v_2} \quad (\text{harmonic progression})$$

Self Practice Problems

Self Practice Problems

1. A person travelling on a straight line moves with a uniform velocity v_1 for some time and with uniform velocity v_2 for the next equal time. The average velocity v is given by

Answer Key : 1. $v = \frac{v_1 + v_2}{2}$ (Arithmetic progression)



(vi) Acceleration :

It is defined as the rate of change of velocity.

It is a vector quantity.

Its direction is same as that of change in velocity and not of the velocity (That is why, acceleration in circular motion is towards the centre).

There are three ways possible in which change in velocity may occur.

When only direction	When only magnitude changes	When both the direction changes and magnitude change
To change the direction, net acceleration or net force should be perpendicular to direction of velocity.	In this case, net force or net acceleration should be parallel or antiparallel to the direction of velocity. (straight line motion)	In this case, net force or net acceleration has two components. One component is parallel or antiparallel to velocity and another one is perpendicular to velocity.
Ex: Uniform circular motion.	Ex: When ball is thrown up under gravity.	Ex: Projectile motion

Types of acceleration -

- a. **Instantaneous acceleration :** It is defined as the acceleration of a body at some particular instant.

$$\text{Instantaneous acceleration} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

- b. **Average acceleration** = $a_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$

- c. **Uniform acceleration :** A body is said to have uniform acceleration if magnitude and direction of the acceleration remains constant during particle motion.

Note : If a particle is moving with uniform acceleration, this does not necessarily imply that particle is moving in straight line.

Example : Parabolic motion

- (d) **Non-uniform acceleration :** A body is said to have non-uniform acceleration, if magnitude or direction or both, change during motion.

Note :

- (i) Acceleration is a vector with dimensions [LT⁻²] and SI units (m/s²)
- (ii) If acceleration is zero, velocity will be constant and motion will be uniform.
- (iii) However if acceleration is constant acceleration is uniform but motion is non-uniform and if acceleration is not constt. both motion and acceleration are non-uniform.

Special note - I

If the motion of a particle is accelerated translatory (without change in direction) $\vec{v} = |\vec{v}| \hat{n}$

$$\Rightarrow \frac{d\vec{v}}{dt} = \frac{d}{dt}[|\vec{v}| \hat{n}] = \hat{n} \frac{d|\vec{v}|}{dt} \quad [\text{as } \hat{n} \text{ is constt.}] \Rightarrow \left| \frac{d\vec{v}}{dt} \right| = \frac{d|\vec{v}|}{dt} (\neq 0)$$

Howeverm, if motion is uniform translatory, both these will still be equal but zero.

Special note - II

$$(i) \quad \frac{d|\vec{v}|}{dt} = 0 \quad \text{while} \quad \left| \frac{d\vec{v}}{dt} \right| \neq 0 \quad (\text{it is possible.})$$

$$(ii) \quad \frac{d|\vec{v}|}{dt} \neq 0 \quad \text{while} \quad \left| \frac{d\vec{v}}{dt} \right| = 0 \quad (\text{it is not possible.})$$

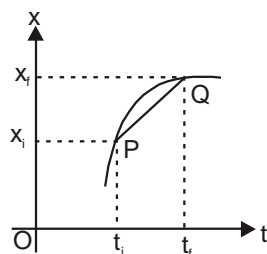
3. GRAPHICAL INTERPRETATION OF SOME QUANTITIES**(i) Average Velocity**

If a particle passes a point P (x_i) at time $t = t_i$ and reaches Q (x_f) at a later time instant

$$t = t_f, \text{ its average velocity in the interval PQ is } V_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$



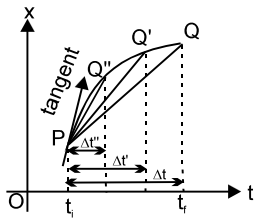
This expression suggests that the average velocity is equal to the slope of the line (chord) joining the points corresponding to P and Q on the x-t graph.

**(ii) Instantaneous Velocity**

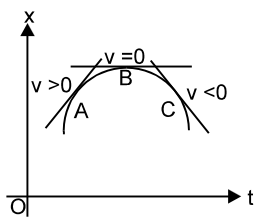
Consider the motion of the particle between the two points P and Q on the x-t graph shown. As the point Q is brought closer and closer to the point P, the time interval between PQ (Δt , $\Delta t'$, $\Delta t''$,.....) get progressively smaller. The average velocity for each time interval is the slope of the appropriate dotted line (PQ, PQ', PQ''.....).

As the point Q approaches P, the time interval approaches zero, but at the same time the slope of the dotted line approaches that of the tangent to the curve at the point P. As $\Delta t \rightarrow 0$, $V_{av} (= \Delta x / \Delta t) \rightarrow V_{inst.}$

Geometrically, as $\Delta t \rightarrow 0$, chord $PQ \rightarrow$ tangent at P.

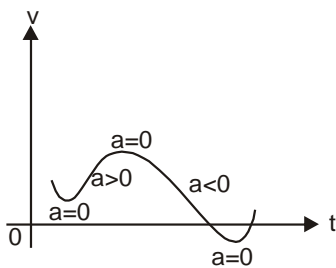


Hence the instantaneous velocity at P is the slope of the tangent at P in the $x - t$ graph. When the slope of the $x - t$ graph is positive, v is positive (as at the point A in figure). At C, v is negative because the tangent has negative slope. The instantaneous velocity at point B (turning point) is zero as the slope is zero.



(iii) Instantaneous Acceleration :

The derivative of velocity with respect to time is the slope of the tangent in velocity time ($v-t$) graph.



Solved Examples

Solved Examples

Example 1. Position of a particle as a function of time is given as $x = 5t^2 + 4t + 3$. Find the velocity and acceleration of the particle at $t = 2$ s?

Solution : Velocity; $v = \frac{dx}{dt} = 10t + 4$

At $t = 2$ s

$$v = 10(2) + 4$$

$$v = 24 \text{ m/s}$$

Acceleration; $a = \frac{d^2x}{dt^2} = 10$

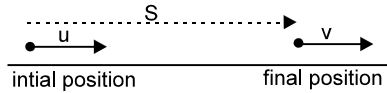
Acceleration is constant, so at $t = 2$ s $a = 10 \text{ m/s}^2$



4. UNIFORMLY ACCELERATED MOTION :

If a particle is accelerated with constant acceleration in an interval of time, then the motion is termed as uniformly accelerated motion in that interval of time.

For uniformly accelerated motion along a straight line (x -axis) during a time interval of t seconds, the following important results can be used.



a) $a = \frac{v - u}{t}$

b) $V_{av} = \frac{v + u}{2}$

c) $S = (v_{av})t$

d) $S = \left(\frac{v + u}{2} \right) t$

e) $v = u + at$

f) $s = ut + \frac{1}{2} at^2$
 $s = vt - \frac{1}{2} at^2$

$$x_f = x_i + ut + \frac{1}{2} at^2$$

$$v^2 = u^2 + 2as$$

g) $s_n = u + a/2 (2n - 1)$

u = initial velocity (at the beginning of interval)

a = acceleration

v = final velocity (at the end of interval)

s = displacement ($x_f - x_i$)

x_f = final coordinate (position)

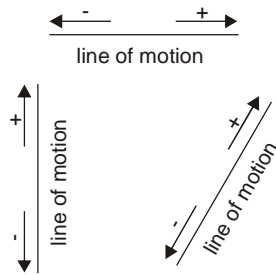
x_i = initial coordinate (position)

s_n = displacement during the n^{th} sec

5. DIRECTIONS OF VECTORS IN STRAIGHT LINE MOTION

In straight line motion, all the vectors (position, displacement, velocity & acceleration) will have only one component (along the line of motion) and there will be only two possible directions for each vector.

For example, if a particle is moving in a horizontal line (x -axis), the two directions are right and left. Any vector directed towards right can be represented by a positive number and towards left can be represented by a negative number. For vertical or inclined motion, upward direction can be taken +ve and downward as -ve



For objects moving vertically near the surface of the earth, the only force acting on the particle is its weight (mg) i.e. the gravitational pull of the earth. Hence acceleration for this type of motion will always be $a = -g$ i.e. $a = -9.8 \text{ m/s}^2$ (-ve sign, because the force and acceleration are directed downwards, If we select upward direction as positive).

Note :

If acceleration is in same direction as velocity, then speed of the particle increases.

If acceleration is in opposite direction to the velocity then speed decreases i.e. the particle slows down.

This situation is known as *retardation*.

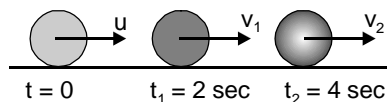
Solved Examples

Solved Examples

Example 1. The velocity acquired by a body moving with uniform acceleration is 20 m/s in first 2 sec and 40 m/s in first 4 sec. Calculate initial velocity.

Solution :
$$a = \frac{v_2 - v_1}{t_2 - t_1}$$

$$a = \frac{40 - 20}{4 - 2} = \frac{20}{2} = 10 \text{ m/s}^2$$



Now, $v = u + at$

$$v_1 = u + at_1$$

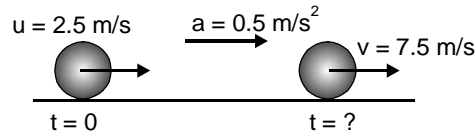
$$\Rightarrow 20 = u + 10 \times 2$$

$$\Rightarrow 20 = u + 20 \quad \Rightarrow \quad u = 0 \text{ m/s} \quad \text{Ans}$$

Example 2. A particle starts with initial velocity 2.5 m/s along the x direction and accelerates uniformly at the rate 50 cm/s². Find time taken to increase the velocity to 7.5 m/s.

Solution : $v = u + at$

$$7.5 = 2.5 + 0.5t$$



$$5.0 = 0.5t$$

$$t = \frac{50}{5} = 10 \text{ sec}$$

Example 3. A particle starts with a constant acceleration. At a time t second speed is found to be 100 m/s and one second later speed becomes 150 m/s. Find acceleration of the particle.

Solution : From equation (1) of motion $v = u + at$

$$100 = 0 + at \quad \Rightarrow \quad 100 = at \quad \dots (1)$$

Now consider velocity one second later -

$$v' = 0 + a(t+1) \quad \Rightarrow \quad 150 = a(t+1) \quad \dots (2)$$

On subtracting equation (1) from equation (2)

$$a = 50 \text{ m/s}^2$$

Example 4. A body travels a distance of 20 m in the 7th second and 24 m in 9th second. How much distance shall it travel in the 15th second?

Solution : Here, $s_7 = 20 \text{ m}$; $s_9 = 24 \text{ m}$,

$$s_{15} = ?$$

Let u and a be the initial velocity and uniform acceleration of the body.

$$\text{We know that, } s_n = u + \frac{a}{2}(2n - 1)$$

$$\therefore s_7 = u + \frac{a}{2}(2 \times 7 - 1) \quad \text{or} \quad 20 = u + \frac{13a}{2} \quad \dots (i)$$

$$\text{and } s_9 = u + \frac{a}{2}(2 \times 9 - 1) \quad \text{or} \quad 24 = u + \frac{17}{2} a \quad \dots (ii)$$

Subtracting (ii) from (i), we get

$$4 = 2a \quad \text{or} \quad a = 2 \text{ ms}^{-2}$$

Putting this value in (i), we get

$$20 = u + \frac{13}{2} \times 2 \quad \text{or} \quad 20 = u + 13$$

or $u = 20 - 13 = 7 \text{ ms}^{-1}$

Hence, $s_{15} = u + \frac{a}{2} (2 \times 15 - 1) = 7 + \frac{2}{2} \times 29$

$s_{15} = 36 \text{ m}$ **Ans.**

Example 5. A person travelling at 43.2 km/h applies the brakes giving a deceleration of 6 m/s^2 to his scooter. How far will it travel before stopping?

Solution : Here, $u = 43.2 \text{ km/h} = 43.2 \times \frac{5}{18} \text{ m/s}$

Deceleration; $a = 6 \text{ m/s}^2$ $v = 0$ $s = ?$

Using $v^2 = u^2 - 2as$

$0 = (12)^2 - 2 \times 6 s$

$144 = 2 \times 6s$

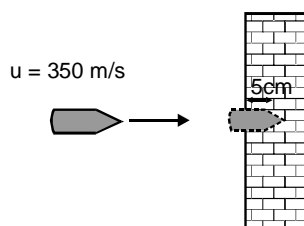
$s = \frac{144}{12} = 12 \text{ m}$ **Ans.**

Example 6. A bullet going with speed 350 m/s enters in a concrete wall and penetrates a distance of 5 cm before coming to rest. Find deceleration.

Solution : Here, $u = 350 \text{ m/s}$, $s = 5 \text{ cm}$, $v = 0 \text{ m/s}$ and $a = ?$

By using $v^2 = u^2 - 2as$

$0 = u^2 - 2as$ $u^2 = 2as$ or $a = \frac{u^2}{2s}$



$a = \frac{350 \times 350}{2 \times 0.05} = 12.25 \times 10^5 \text{ m/sec}^2$

Example 7. A driver takes 0.20 s to apply the brakes after he sees a need for it. This is called the reaction time of the driver. If he is driving a car at a speed of 54 km/h and the brakes cause a deceleration of 6.0 m/s^2 , find the distance travelled by the car after he sees the need to put the brakes on.

Solution : Distance covered by the car during the application of brakes by driver -

$u = 54 \text{ km/h} = 54 \times \frac{5}{18} \text{ m/s} = 15 \text{ m/s}$

$s_1 = ut$ or $s_1 = 15 \times 0.2 = 3.0 \text{ meter}$

After applying the brakes;

$$v = 0 \quad u = 15 \text{ m/s}, \quad a = 6 \text{ m/s}^2 \quad s_2 = ?$$

Using $v^2 = u^2 - 2as$

$$0 = (15)^2 - 2 \times 6 \times s_2$$

$$12 s_2 = -225$$

$$\Rightarrow s_2 = \frac{225}{12} = 18.75 \text{ metre}$$

Distance travelled by the car after driver sees the need for it

$$s = s_1 + s_2$$

$$s = 3 + 18.75 = 21.75 \text{ metre.} \quad \text{Ans.}$$

Self Practice Problems

Self Practice Problems

Example 1. A particle moving rectilinearly with constant acceleration is having initial velocity of 10 m/s. After some time, its velocity becomes 30 m/s. Find out velocity of the particle at the mid point of its path?

Solution : Let the total distance be $2x$.

$$\therefore \text{ distance upto midpoint} = x$$

Let the velocity at the mid point be v

and acceleration be a .

From equations of motion

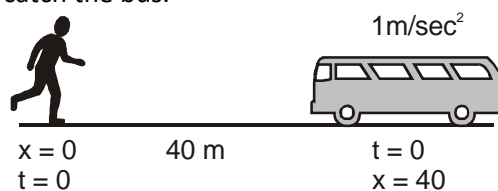
$$v^2 = 10^2 + 2ax \quad \text{--- (1)}$$

$$30^2 = v^2 + 2ax \quad \text{--- (2)}$$

(2) - (1) gives

$$v^2 - 30^2 = 10^2 - v^2 \Rightarrow v^2 = 500 \quad \Rightarrow v = 10\sqrt{5} \text{ m/s}$$

Example 2. A man is standing 40 m behind the bus. Bus starts with 1 m/sec^2 constant acceleration and also at the same instant the man starts moving with constant speed 9 m/s. Find the time taken by man to catch the bus.



Solution : Let after time 't' man will catch the bus

For bus

$$x = x_0 + ut + \frac{1}{2} at^2, \quad x = 40 + 0(t) + \frac{1}{2} (1) t^2$$

$$x = 40 + \frac{t^2}{2} \quad \dots\dots\dots (i)$$

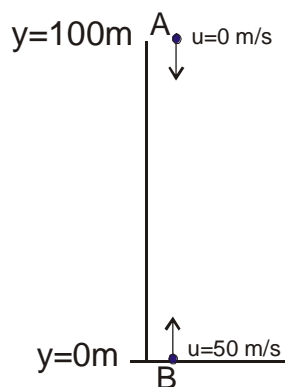
$$\text{For man, } x = 9t \quad \dots\dots\dots (ii)$$

From (i) & (ii)

$$40 + \frac{t^2}{2} = 9t \quad \text{or} \quad t = 8 \text{ s or } t = 10\text{s.}$$

Example 3. A particle is dropped from height 100 m and another particle is projected vertically up with velocity 50 m/s from the ground along the same line. Find out the position where two particle will meet?

Solution : Let the upward direction is positive.



Let the particles meet at a distance y from the ground.

For particle A,

$$y_0 = + 100 \text{ m}$$

$$u = 0 \text{ m/s}$$

$$a = - 10 \text{ m/s}^2$$

$$y = 100 + 0(t) - \frac{1}{2} \times 10 \times t^2 \quad [y = y_0 + ut + \frac{1}{2} at^2]$$

$$= 100 - 5t^2 \quad \text{---- (1)}$$

For particle B,

$$y_0 = 0 \text{ m}$$

$$u = + 50 \text{ m/s}$$

$$a = - 10 \text{ m/s}^2$$

$$y = 50(t) - \frac{1}{2} \times 10 \times t^2$$

$$= 50t - 5t^2 \quad \text{---- (2)}$$

According to the problem;

$$50t - 5t^2 = 100 - 5t^2$$

$$t = 2 \text{ sec}$$

Putting $t = 2$ sec in eqn. (1),

$$y = 100 - 20 = 80 \text{ m}$$

Hence, the particles will meet at a height 80 m above the ground.

Example 4. A particle is dropped from a tower. It is found that it travels 45 m in the last second of its journey. Find out the height of the tower?

Solution : Let the total time of journey be n seconds.

$$\text{Using; } s_n = u + \frac{a}{2}(2n-1) \quad \Rightarrow \quad 45 = 0 + \frac{10}{2}(2n-1)$$

$$n = 5 \text{ sec}$$

$$\text{Height of tower; } h = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times 5^2 = 125 \text{ m}$$

Example 5. A ball is projected vertically up with an initial speed of 20 m/s on a planet where acceleration due to gravity is 10 m/s^2

- (a) How long does it takes to reach the highest point?
- (b) How high does it rise above the point of projection?
- (c) How long will it take for the ball to reach a point 10 m above the point of projection?

Solution : As here motion is vertically upwards,

$$a = -g \text{ and } v = 0$$

(a) From 1st equation of motion, i.e., $v = u + at$, $0 = 20 - 10t$ i.e. $t = 2$ sec.
Ans.

(b) Using $v^2 = u^2 + 2as$ $0 = (20)^2 - 2 \times 10 \times h$ i.e. $h = 20$ m. **Ans.**

(c) Using $s = ut + \frac{1}{2}at^2$,
 $10 = 20t - \frac{1}{2} \times 10 \times t^2$

$$\text{i.e. } t^2 - 4t + 2 = 0$$

$$\text{or } t = 2 \pm \sqrt{2},$$

$$\text{i.e. } t = 0.59 \text{ sec. or } 3.41 \text{ sec.}$$

i.e., there are two times, at which the ball passes through $h = 10$ m, once while going up and then coming down.



Example 6. A ball is thrown vertically upwards from a bridge with an initial velocity of 4.9 m/s. It strikes the water after 2s. If acceleration due to gravity is 9.8 m/s² (a) What is the height of the bridge? (b) With which velocity does the ball strike the water?

Solution : Taking the point of projection as origin and downward direction as positive,

(a) Using $s = ut + \left(\frac{1}{2}\right)at^2$ we have

$$h = -4.9 \times 2 + \left(\frac{1}{2}\right)9.8 \times 2^2 = 9.8\text{m} \quad (\text{u is taken to be negative as it is upwards})$$

(b) Using $v = u + at$
 $v = -4.9 + 9.8 \times 2 = 14.7 \text{ m/s}$

Example 7. A rocket is fired vertically up from the ground with a resultant vertical acceleration of 10 m/s². The fuel is finished in 1minute and it continues to move up.

(a) What is the maximum height reached?

(b) After how much time from then will the maximum height be reached? (Take $g = 10 \text{ m/s}^2$)

Solution : (a) The distance travelled by the rocket during burning interval (1 minute = 60 s) in which resultant acceleration is vertically upwards and 10 m/s² will be

$$h_1 = 0 \times 60 + (1/2) \times 10 \times 60^2 = 18000\text{m} \quad \dots \quad (1)$$

Velocity acquired by it is

$$v = 0 + 10 \times 60 = 600 \text{ m/s} \quad \dots \quad (2)$$

After one minute the rocket moves vertically up with initial velocity of 600 m/s and continues till height h_2 till its velocity becomes zero.

$$0 = (600)^2 - 2gh_2$$

$$\text{or} \quad [\text{as } g = 10 \text{ m/s}^2] \quad \dots \quad (3)$$

From equations (1) and (3) the maximum height reached by the rocket from the ground is

$$H = h_1 + h_2 = 18 + 18 = 36\text{km}$$

(b) The time to reach maximum height after burning of fuel is

$$0 = 600 - gt \quad t = 60 \text{ s}$$

After finishing fuel the rocket goes up for 60 s.

Example 8. A body is released from a height and falls freely towards the earth. Exactly 1 sec later another body is released. What is the distance between the two bodies after 2 sec the release of the second body, if $g = 9.8 \text{ m/s}^2$

Solution : The 2nd body falls for 2s, so

$$h_2 = \frac{1}{2}g(2)^2 \quad \dots \quad (1)$$

While 1st has fallen for $2 + 1 = 3$ sec so

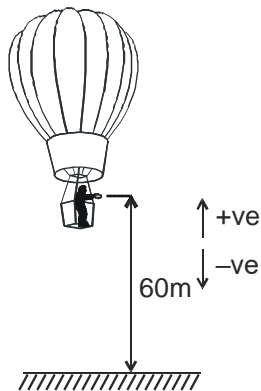
$$h_1 = \frac{1}{2}g(3)^2 \quad \dots \quad (2)$$

∴ Separation between two bodies after 2 sec the release of 2nd body,

$$d = h_1 - h_2 = \frac{1}{2}g(3^2 - 2^2) = 4.9 \times 5 = 24.5\text{m}$$

Example 9. A stone is dropped from a balloon going up with a uniform velocity of 5 m/s. If the balloon was 60 m high when the stone was dropped, find its height when the stone hits the ground. Take $g = 10 \text{ m/s}^2$.

Solution : $S = ut + \frac{1}{2} at^2 - 60 = 5(t) + \frac{1}{2} (-10) t^2$



$$-60 = 5t - 5t^2 \quad \Rightarrow \quad 5t^2 - 5t - 60 = 0$$

$$t^2 - t - 12 = 0 \quad \Rightarrow \quad t^2 - 4t + 3t - 12 = 0$$

$$(t - 4)(t + 3) = 0 \Rightarrow \quad t = 4$$

Height of balloon from ground at this instant

$$= 60 + 4 \times 5 = 80 \text{ m}$$

Note : As the particle is detached from the balloon it is having the same velocity as that of balloon, but its acceleration is only due to gravity and is equal to g .

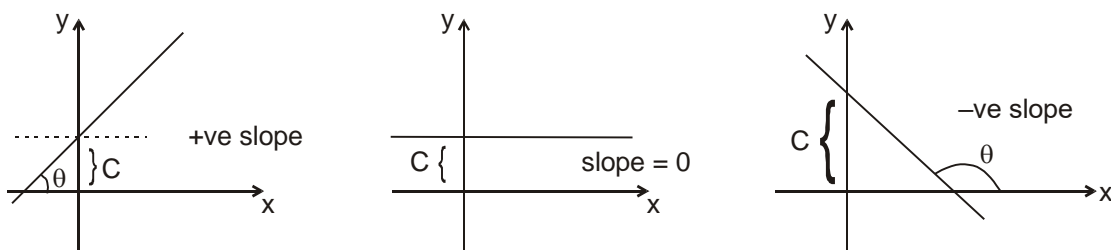


6. STRAIGHT LINE-EQUATION, GRAPH, SLOPE (+VE, -VE, ZERO SLOPE).

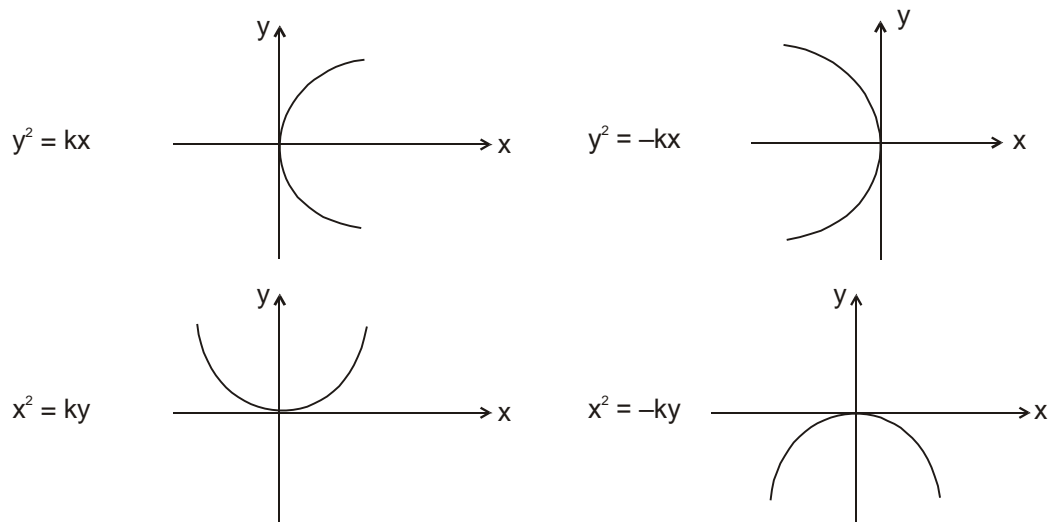
If θ is the angle at which a straight line is inclined to the positive direction of x -axis, & $0^\circ \leq \theta < 180^\circ$, $\theta \neq 90^\circ$, then the slope of the line, denoted by m , is defined by $m = \tan \theta$. If θ is 90° , m does not exist, but the line is parallel to the y -axis. If $\theta = 0$, then $m = 0$ & the line is parallel to the x -axis.

Slope – intercept form : $y = mx + c$ is the equation of a straight line whose slope is m & which makes an intercept c on the y -axis.

$$m = \text{slope} = \tan \theta = \frac{dy}{dx}$$



7. PARABOLIC CURVE-EQUATION, GRAPH (VARIOUS SITUATIONS UP, DOWN, LEFT, RIGHT WITH CONDITIONS)

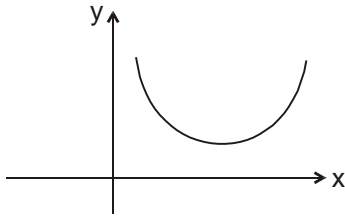


Where k is a positive constant.

Equation of parabola :

Case (i) : $y = ax^2 + bx + c$

For $a > 0$

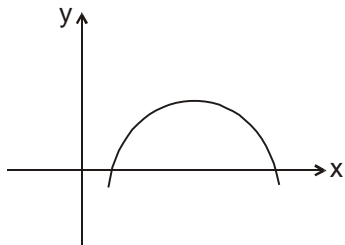


The nature of the parabola will be like that of the nature $x^2 = ky$

Minimum value of y exists at the vertex of the parabola.

$$y_{\min} = \frac{-D}{4a} \text{ where } D = b^2 - 4ac$$

$$\text{Coordinates of vertex} = \left(\frac{-b}{2a}, \frac{D}{4a} \right)$$

Case (ii) : $a < 0$ 

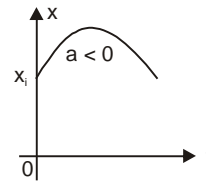
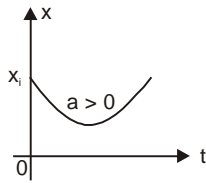
The nature of the parabola will be like that of the nature of $x^2 = -ky$

Maximum value of y exists at the vertex of parabola.

$$y_{\max} = \frac{D}{4a} \text{ where } D = b^2 - 4ac$$

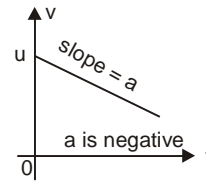
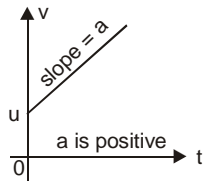
8. GRAPHS IN UNIFORMLY ACCELERATED MOTION ($A \neq 0$)

- x is a quadratic polynomial in terms of t . Hence $x-t$ graph is a parabola.



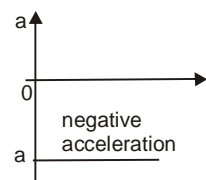
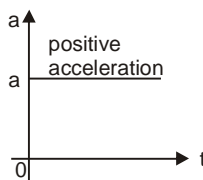
x-t graph

- v is a linear polynomial in terms of t . Hence $v-t$ graph is a straight line of slope a .



v-t graph

- $a-t$ graph is a horizontal line because a is constant.



a-t graph

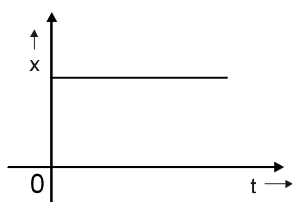
9. INTERPRETATION OF SOME MORE GRAPHS

9.1. Position Vs Time graph

(i) Zero Velocity

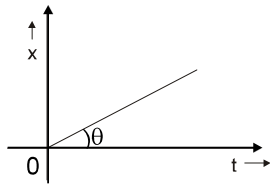
As position of particle is fixed at all the time,
so the body is at rest.

$$\text{Slope; } \frac{dx}{dt} = \tan\theta = \tan 0^\circ = 0$$



Velocity of particle is zero

(ii) Uniform Velocity



Here $\tan\theta$ is constant $\tan\theta = \frac{dx}{dt}$

$\therefore \frac{dx}{dt}$ is constant.

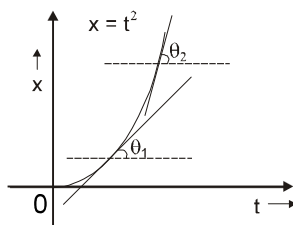
\therefore velocity of particle is constant.

Non uniform velocity (increasing with time)

In this case;

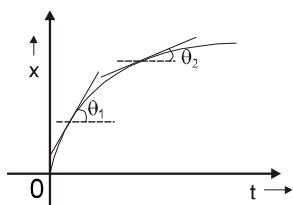
As time is increasing, θ is also increasing.

$\therefore \frac{dx}{dt} = \tan\theta$ is also increasing



Hence, velocity of particle is increasing.

Non uniform velocity (decreasing with time)



In this case;

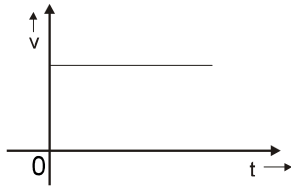
As time increases, θ decreases.

$\therefore \frac{dx}{dt} = \tan\theta$ also decreases.

Hence, velocity of particle is decreasing.

9.2. Velocity vs time graph

(i) Zero acceleration



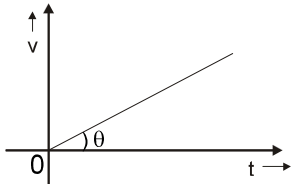
Velocity is constant.

$$\tan\theta = 0$$

$$\therefore \frac{dv}{dt} = 0$$

Hence, acceleration is zero.

(ii) Uniform acceleration

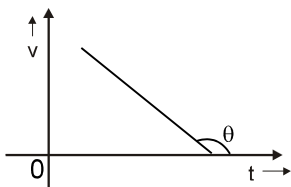


$\tan\theta$ is constant.

$$\therefore \frac{dv}{dt} = \text{constant}$$

Hence, it shows constant acceleration.

(iii) Uniform retardation



Since $\theta > 90^\circ$

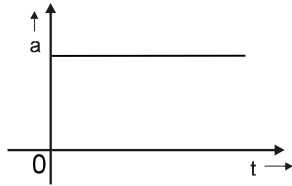
$\therefore \tan\theta$ is constant and negative.

$$\therefore \frac{dv}{dt} = \text{negative constant}$$

Hence, it shows constant retardation.

(iv) Acceleration vs time graph

a. Constant acceleration



$$\tan\theta = 0$$

$$\therefore \frac{da}{dt} = 0$$

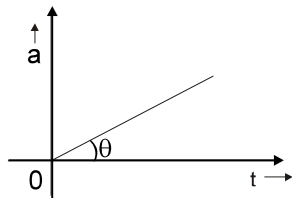
Hence, acceleration is constant.

(b) Uniformly increasing acceleration

θ is constant.

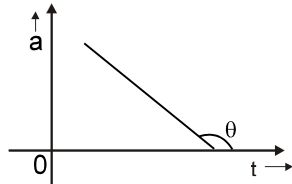
$$0^\circ < \theta < 90^\circ \Rightarrow \tan\theta > 0$$

$$\therefore \frac{da}{dt} = \tan\theta = \text{constant} > 0$$



Hence, acceleration is uniformly increasing with time.

(c) Uniformly decreasing acceleration



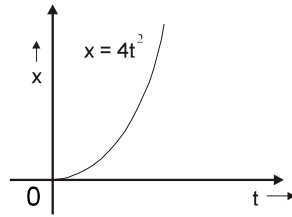
Since $\theta > 90^\circ$

$\therefore \tan\theta$ is constant and negative.

$$\therefore \frac{da}{dt} = \text{negative constant}$$

Hence, acceleration is uniformly decreasing with time

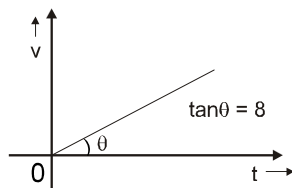
Example 10. The displacement vs time graph of a particle moving along a straight line is shown in the figure. Draw velocity vs time and acceleration vs time graph.



Solution : $x = 4t^2$

$$v = \frac{dx}{dt} = 8t$$

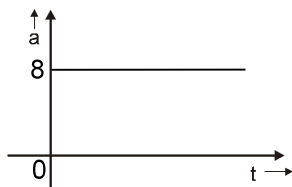
Hence, velocity-time graph is a straight line



having slope i.e. $\tan\theta = 8$.

$$a = \frac{dv}{dt} = 8$$

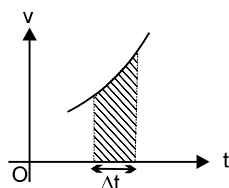
Hence, acceleration is constant throughout and is equal to 8.



10. DISPLACEMENT FROM V - T GRAPH & CHANGE IN VELOCITY FROM A -T GRAPH

Displacement = Δx = area under v-t graph.

Since a negative velocity causes a negative displacement, areas below the time axis are taken negative. In similar way, can see that $\Delta v = a \Delta t$ leads to the conclusion that **area under a - t graph gives the change**

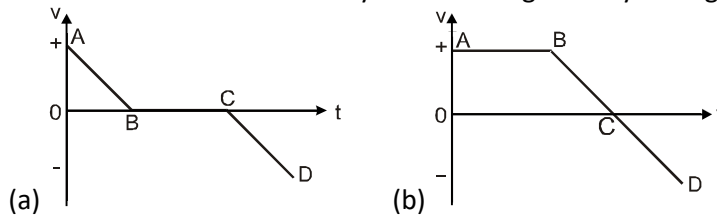


in velocity Δv during that interval.

Solved Examples

Solved Examples

Example 1. Describe the motion shown by the following velocity-time graphs.



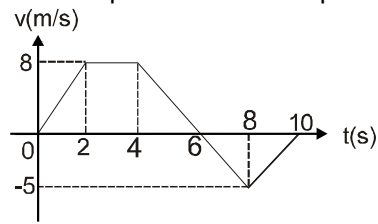
Solution : (a) **During interval AB:** velocity is +ve so the particle is moving in +ve direction, but it is slowing down as acceleration (slope of v-t curve) is negative. **During interval BC:** particle remains at rest as velocity is zero. Acceleration is also zero. **During interval CD:** velocity is -ve so the particle is moving in -ve direction and is speeding up as acceleration is also negative.

(b) **During interval AB:** particle is moving in +ve direction with constant velocity and acceleration is zero. **During interval BC:** particle is moving in +ve direction as velocity is +ve, but it slows down until it comes to rest as acceleration is negative. **During interval CD:** velocity is -ve so the particle is moving in -ve direction and is speeding up as acceleration is also negative.

Important Points to Remember

- For uniformly accelerated motion ($a \neq 0$), $x-t$ graph is a parabola (opening upwards if $a > 0$ and opening downwards if $a < 0$). The slope of tangent at any point of the parabola gives the velocity at that instant.
- For uniformly accelerated motion ($a \neq 0$), $v-t$ graph is a straight line whose slope gives the acceleration of the particle.
- In general, the slope of tangent in $x-t$ graph is velocity and the slope of tangent in $v-t$ graph is the acceleration.
- The area under $a-t$ graph gives the change in velocity.
- The area under the $v-t$ graph gives the distance travelled by the particle, if we take all areas as positive.
- Area under $v-t$ graph gives displacement, if areas below the t -axis are taken negative.

Example 2. For a particle moving along x-axis, velocity-time graph is as shown in figure. Find the distance travelled and displacement of the particle?



Solution : Distance travelled = Area under v-t graph (taking all areas as +ve.)

Distance travelled = Area of trapezium + Area of triangle

$$= \frac{1}{2}(2+6) \times 8 + \frac{1}{2} \times 4 \times 5 = 32 + 10 = 42 \text{ m}$$

Displacement = Area under v-t graph (taking areas below time axis as -ve.)

Displacement = Area of trapezium – Area of triangle

$$= \frac{1}{2}(2+6) \times 8 - \frac{1}{2} \times 4 \times 5 = 32 - 10 = 22 \text{ m}$$

Hence, distance travelled = 42 m and displacement = 22 m.

11. MOTION WITH NON-UNIFORM ACCELERATION (USE OF DEFINITE INTEGRALS)

$$\Delta x = \int_{t_i}^{t_f} v(t) dt \text{ (displacement in time interval } t = t_i \text{ to } t_f)$$

The expression on the right hand side is called the *definite integral* of $v(t)$ between $t = t_i$ and $t = t_f$. Similarly change in velocity

$$\Delta v = v_f - v_i = \int_{t_i}^{t_f} a(t) dt$$

Solving Problems which Involves Nonuniform Acceleration

Acceleration depending on velocity v or time t

By definition of acceleration, we have $a = \frac{dv}{dt}$. If a is in terms of t ,

$$\int_{v_0}^v dv = \int_0^t a(t) dt. \text{ If } a \text{ is in terms of } v, \int_{v_0}^v \frac{dv}{a(v)} = \int_0^t dt.$$

On integrating, we get a relation between v and t , and then

$$\text{using } \int_{x_0}^x dx = \int_0^t v(t) dt, x \text{ and } t \text{ can also be related.}$$

Acceleration depending on velocity v or position x

$$a = \frac{dv}{dt} \quad \Rightarrow \quad a = \frac{dv}{dx} \frac{dx}{dt} \quad \Rightarrow \quad a = \frac{dx}{dt} \frac{dv}{dx} \quad \Rightarrow \quad a = v \frac{dv}{dx}$$

This is another important expression for acceleration.

If a is in terms of x,

$$\int_{v_0}^v v \, dv = \int_{x_0}^x a(x) \, dx.$$

If a is in terms of v, $\int_{v_0}^v \frac{v \, dv}{a(v)} = \int_{x_0}^x dx$

On integrating, we get a relation between x and v.

Using $\int_{x_0}^x \frac{dx}{v(x)} = \int_0^t dt$, we can relate x and t.

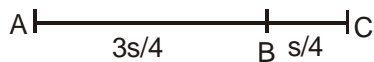
Solved Miscellaneous Problems

Solved Miscellaneous Problems

Problem 1. A particle covers $\frac{3}{4}$ of total distance with speed v_1 and next $\frac{1}{4}$ with v_2 . Find the average speed of the particle?

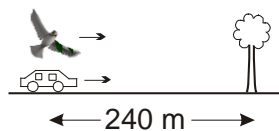
Answer : $\frac{4v_1v_2}{v_1 + 3v_2}$

Solution : Let the total distance be s



$$\text{average speed } \langle v \rangle = \frac{\text{Total distance}}{\text{Total time taken}} \quad \langle v \rangle = \frac{s}{\frac{3s}{4v_1} + \frac{s}{4v_2}} = \frac{1}{\frac{3}{4v_1} + \frac{1}{4v_2}} = \frac{4v_1v_2}{v_1 + 3v_2}$$

Problem 2. A car is moving with speed 60 Km/h and a bird is moving with speed 90 km/h along the same direction as shown in figure. Find the distance travelled by the bird till the time car reaches the tree?



Answer : 360 m

Solution : Time taken by a car to reaches the tree (t) = $\frac{240 \text{ m}}{60 \text{ km/hr}} = \frac{0.24}{60} \text{ hr}$

Now, the distance travelled by the bird during this time interval (s)

$$= 90 \times \frac{0.24}{60} = 0.12 \times 3 \text{ km} = 360 \text{ m}.$$

Problem 3. The position of a particle moving on X-axis is given by

$$x = At^3 + Bt^2 + Ct + D.$$

The numerical values of A, B, C, D are 1, 4, -2 and 5 respectively and SI units are used. Find (a) the dimensions of A, B, C and D, (b) the velocity of the particle at $t = 4$ s, (c) the acceleration of the particle at $t = 4$ s, (d) the average velocity during the interval $t = 0$ to $t = 4$ s, (e) the average acceleration during the interval $t = 0$ to $t = 4$ s.

Answer : (a) $[A] = [LT^{-3}]$, $[B] = [LT^{-2}]$, $[C] = [LT^{-1}]$ and $[D] = [L]$;

(b) 78 m/s ; (c) 32 m/s² ; (d) 30 m/s ; (e) 20 m/s²

Solution : As $x = At^3 + Bt^2 + Ct + D$

(a) Dimensions of A, B, C and D ,

$[At^3] = [x]$ (by principle of homogeneity)

$[A] = [LT^{-3}]$ similarly, $[B] = [LT^{-2}]$, $[C] = [LT^{-1}]$ and $[D] = [L]$;

(b) As $v = \frac{dx}{dt} = 3At^2 + 2Bt + C$

velocity at $t = 4$ sec.

$$v = 3(1)(4)^2 + 2(4)(4) - 2 = 78 \text{ m/s.}$$

(c) Acceleration $(a) = \frac{dv}{dt} = 6At + 2B$; $a = 32 \text{ m/s}^2$

(d) average velocity as $x = At^3 + Bt^2 + Ct + D$

position at $t = 0$, is $x = D = 5 \text{ m}$.

Position at $t = 4$ sec is $(1)(64) + (4)(16) - (2)(4) + 5 = 125 \text{ m}$

Thus the displacement during 0 to 4 sec. is $125 - 5 = 120 \text{ m}$

$$\therefore \langle v \rangle = 120 / 4 = 30 \text{ m/s}$$

(e) $v = 3At^2 + 2Bt + C$, velocity at $t = 0$ is $c = -2 \text{ m/s}$

$$\text{velocity at } t = 4 \text{ sec is } 78 \text{ m/s} \quad \therefore \langle a \rangle = \frac{v_2 - v_1}{t_2 - t_1} = 20 \text{ m/s}^2$$

Problem 4. A car decelerates from a speed of 20 m/s to rest in a distance of 100 m. What was its acceleration, assumed constant?

Solution : $v = 0$ $u = 20 \text{ m/s}$ $s = 100 \text{ m}$ \Rightarrow as $v^2 = u^2 + 2as$

$$0 = 400 + 2a \times 100 \quad \Rightarrow \quad a = -2 \text{ m/s}^2$$

\therefore acceleration = **2m/s²** **Ans.**

Problem 5. A 150 m long train accelerates uniformly from rest. If the front of the train passes a railway worker 50 m away from the station at a speed of 25 m/s, what will be the speed of the back part of the train as it passes the worker?

Solution : $v^2 = u^2 + 2as \Rightarrow 25 \times 25 = 0 + 100 a \Rightarrow a = \frac{25}{4} \text{ m/s}^2$

Now, for time taken by the back end of the train to pass the worker

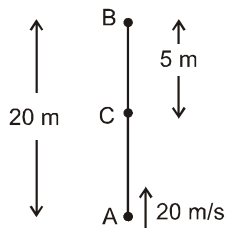
we have $v'^2 = v^2 + 2al = (25)^2 + 2 \times \frac{25}{4} \times 150$

$v'^2 = 25 \times 25 \times 4 \quad v' = 50 \text{ m/s.} \quad \text{Ans.}$

Problem 6. A particle is thrown vertically with velocity 20 m/s . Find (a) the distance travelled by the particle in first 3 seconds, (b) displacement of the particle in 3 seconds.

Answer : 25m, 15m

Solution : Highest point say B



$V_B = 0$

$v = u + gt$

$0 = 20 - 10 t$

$t = 2 \text{ sec.}$

\therefore distance travel in first 2 seconds.

$s = s(t - 0 \text{ to } t = 2 \text{ sec}) + 5 (2 \text{ sec. to } 3 \text{ sec.})$

$s = [ut + 1/2 at^2]_{t=0 \text{ to } t=2s} + [ut + 1/2 at^2]_{t=2 \text{ to } t=3s}$

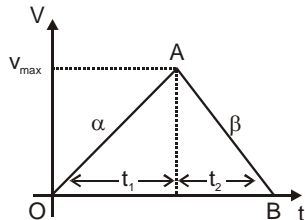
$s = 20 \times 2 - 1/2 \times 10 \times 4 + 1/2 \times 10 \times 1^2$

$= (40 - 20) + 5 = 25 \text{ m. and displacement} = 20 - 5 = 15 \text{ m.}$

Problem 7. A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t . Find the maximum velocity acquired by the car.

Solution : $t = t_1 + t_2$

$$\text{slope of OA curve} = \tan\theta = \alpha = \frac{v_{\max}}{t_1}$$



$$\text{slope of AB curve} = \beta = \frac{v_{\max}}{t_2}$$

$$t = t_1 + t_2 \quad \Rightarrow \quad t = \frac{v_{\max}}{\alpha} + \frac{v_{\max}}{\beta} \quad \Rightarrow \quad v_{\max} = \left(\frac{\alpha + \beta}{\alpha\beta} \right) t$$

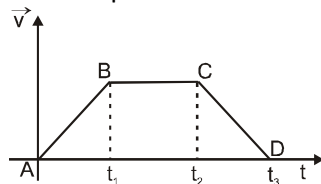
Problem 8. In the above question find total distance travelled by the car in time 't'.

Solution : $v_{\max} = \frac{\alpha + \beta}{\alpha\beta} t \Rightarrow t_1 = \frac{v_{\max}}{\alpha} = \frac{\beta t}{\alpha + \beta} \Rightarrow t_2 = \frac{v_{\max}}{\beta} = \frac{\alpha t}{\alpha + \beta}$

\therefore Total distance travelled by the car in time 't'

$$= \frac{1}{2} \alpha t_1^2 + v_{\max} t_2 - \frac{1}{2} \beta t_2^2 = \frac{1}{2} \frac{\alpha \beta^2 t^2}{(\alpha + \beta)^2} + \frac{\alpha^2 \beta t^2}{(\alpha + \beta)^2} - \frac{1}{2} \frac{\beta \alpha^2 t^2}{(\alpha + \beta)^2} = \frac{1}{2} \frac{\alpha \beta t^2}{(\alpha + \beta)} = \frac{\alpha \beta t^2}{2(\alpha + \beta)} \text{ Ans.}$$

Problem 9. Draw displacement–time and acceleration–time graph for the given velocity–time graph.



Solution : **Part AB :** v-t curve shows constant slope

i.e. constant acceleration or Velocity increases at constant rate with time.

Hence, s-t curve will show constant increase in slope

and a-t curve will be a straight line.

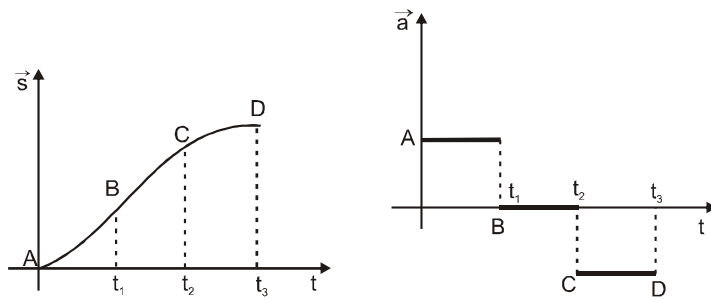
Part BC : v-t curve shows zero slope i.e. constant velocity. So, s-t curve will show constant slope and acceleration will be zero.

Part CD : v-t curve shows negative slope i.e. velocity is decreasing with time or acceleration is negative.

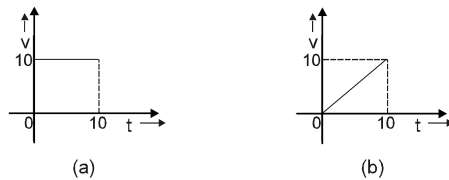
Hence, s-t curve will show decrease in slope becoming zero in the end.

and a-t curve will be a straight line with negative intercept on y-axis.

RESULTING GRAPHS ARE:



Problem 10. For a particle moving along x-axis, following graphs are given. Find the distance travelled by the particle in 10 s in each case?



Solution :

(a) Distance area under the $v - t$ curve
 \therefore distance = $10 \times 10 = 100$ m **Ans.**

(b) Area under $v - t$ curve
 \therefore distance = $\frac{1}{2} \times 10 \times 10 = 50$ m **Ans.**

Problem 11. If the displacement of a particle is $(2t^2 + t + 5)$ meter then, what will be acceleration at $t = 5$ sec.

Solution $v = \frac{dx}{dt} = \frac{d}{dt} (2t^2 + t + 5) = 4t + 1$ m/s and
 $a = \frac{dv}{dt} = \frac{d}{dt} (4t + 1) \quad a = 4$ m/s $a = 4$ m/s²

12. KEY CONCEPTS

☞ **Average Velocity (in an interval) :** $\vec{v}_{av} = \langle \vec{v} \rangle = \frac{\text{Total displacement}}{\text{Total time taken}} = \frac{\vec{r}_f - \vec{r}_i}{\Delta t}$

☞ **Average Speed (in an interval) :** $\text{Average Speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$

☞ **Instantaneous Velocity (at an instant) :** $\vec{v}_{inst} = \lim_{\Delta t \rightarrow 0} \left(\frac{\Delta \vec{r}}{\Delta t} \right) = \frac{d\vec{r}}{dt}$

☞ **Average Acceleration (in an interval):** $\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$

☞ **Instantaneous Acceleration (at an instant):** $\vec{a} = \lim_{\Delta t \rightarrow 0} \left(\frac{\Delta \vec{v}}{\Delta t} \right) = \frac{d\vec{v}}{dt}$

☞ **Equations of Motion (for constant acceleration) :**

(a) $v = u + at$ (b) $s = ut + \frac{1}{2} at^2$; $s = vt - \frac{1}{2} at^2$; $x_f = x_i + ut + \frac{1}{2} at^2$

(c) $v^2 = u^2 + 2as$ (d) $s = \frac{(u+v)}{2} t$

(e) $s_n = u + \frac{a}{2} (2n - 1)$

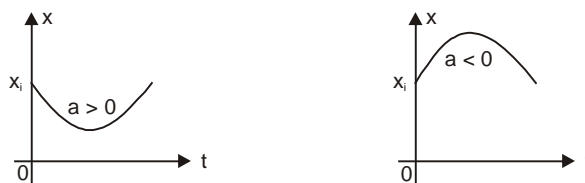
☞ **For freely falling bodies : (u = 0) (taking upward direction as positive)**

(a) $v = -gt$ (b) $s = -\frac{1}{2} gt^2$; $s = vt + \frac{1}{2} gt^2$; $h_f = h_i - \frac{1}{2} gt^2$

(c) $v^2 = -2gs$ (d) $s_n = -\frac{a}{2} (2n - 1)$

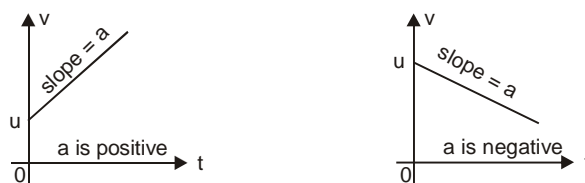
☞ **Graphs in Uniformly Accelerated Motion along a straight line (a ≠ 0) :**

☞ x is a quadratic polynomial in terms of t. Hence x – t graph is a parabola.



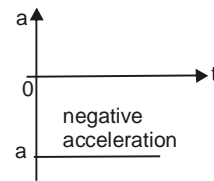
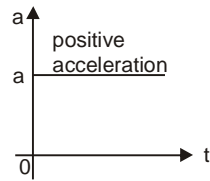
x-t graph

☞ v is a linear polynomial in terms of t. Hence v–t graph is a straight line of slope a.



v-t graph

☞ a–t graph is a horizontal line because a is constant.

**a-t graph**

- ☞ For uniformly accelerated motion ($a \neq 0$), $x-t$ graph is a parabola (opening upwards if $a > 0$ and opening downwards if $a < 0$). The slope of tangent at any point of the parabola gives the velocity at that instant.
- ☞ For uniformly accelerated motion ($a \neq 0$), $v-t$ graph is a straight line whose slope gives the acceleration of the particle.
- ☞ In general, the slope of tangent in $x-t$ graph is velocity and the slope of tangent in $v-t$ graph is the acceleration.
- ☞ The area under $a-t$ graph gives the change in velocity.
- ☞ The area under the $v-t$ graph gives the distance travelled by the particle, if we take all areas as positive.
- ☞ Area under $v-t$ graph gives displacement, if areas below the t -axis are taken negative.

